

## PASIVE SUSPENSION MODELING USING MATLAB, QUARTER CAR MODEL, INPUT SIGNAL STEP TYPE

Andronic Florin<sup>1</sup>, Manolache-Rusu Ioan-Cozmin<sup>2</sup>, Pătuleanu Liliana<sup>3</sup>

<sup>1,2,3</sup> „Ștefan cel Mare” University of Suceava, 13 Universității, 720229, Suceava, Romania,  
florin\_andronic@darex.ro

**Abstract:** *The purpose of a vehicle suspension system is to improve ride comfort and road handling. In current article is simulated and analyzed the handling and ride performance of a vehicle with passive suspension system, quarter car model with two degree of freedom. Since, the equations of the system cannot be solved mathematically has developed a scheme in Matlab Simulink that allows analyzing the behavior of the suspension. The schema that was created in Matlab Simulink, were compared with the State space model and the Transfer function. After completing the simulation scheme can be introduced excitation signals, this case a step signal.*

**Keywords:** *Automotive Suspension, Matlab, Transfer function, State space model*

### 1. Introduction

The vehicle suspension system differ depending on the manufacturer which ensures a wide range of models. Whichever solution is adopted to design, a suspension system has the primary role to ensuring the safety function. It is known that road unevenness produce oscillations of the vehicle wheels which will transmitted to their axles. It becomes clear that the role of the suspension system witch connect the axles to the car body is to reduce as much vibrations and shocks occurring in the operation. This causes, the necessity to using a suspension of a better quality.

A quality suspension must achieve a good behavior of the vehicle and a degree of comfort depending on the interaction with uneven road surface [1, 6]. When the vehicle is requested by uneven road profile, it should not be too large oscillations, and if this occurs, they must be removed as quickly. The design of a vehicle suspension is an issue that requires a series of calculations based on the purpose.

Suspension systems are classified in the well-known terms of passive, semi-active, active and various in between systems. Typical features are the required energy and the characteristic

frequency of the actuator[11]. Passive system are the most common.

So far, several models have been developed [3, 7-10], such as *quarter car*, *half car* or *full car* suspension. The following references will be made to the model suspension *quarter car*, passive suspension system.

### 2. The mathematical model of quarter car suspension

The system shown in Fig.1 is an quarter car system were  $m_1$  - is the sprung mass,  $m_2$  - is the unsprung mass,  $k_1$  - is the stiffness coefficient of the suspension,  $k_2$  - is the vertical stiffness of the tire,  $b_1$  - is the damping coefficient of the suspension,  $b_2$  - is the damping coefficient of the tire,  $x_1$  - the vertical displacement of sprung mass,  $x_2$  - is the vertical displacement of unsprung mass,  $w$  - is the road excitation. We will consider only mass movements on the vertical axis ignoring the rotational movement of the vehicle.

Since the distance  $x_1-w$  is hard to measure and the deformation of the tire  $x_2-w$  is negligible, result that we can use as an input size, the displacement  $x_1-x_2$  against which we will analyze the behavior of the suspension system.

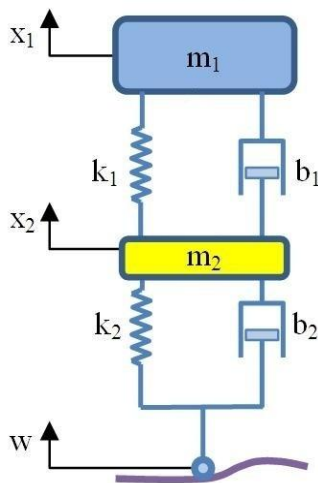


Figure 1 The passive suspension system

The equations of motion can be obtained using the Newton's second law for each of the two masses are in motion and Newton's third law of their interaction . These will be:

$$m_1 \ddot{x}_1 + b_1 (\dot{x}_1 - \dot{x}_2) + k_1 (x_1 - x_2) = 0 \quad (1)$$

$$m_2 \ddot{x}_2 + b_1 (\dot{x}_1 - \dot{x}_2) + k_1 (x_2 - x_1) + b_2 \dot{x}_2 + k_2 x_2 = 0 \quad (2)$$

Separate terms  $\ddot{x}_1$  and  $\ddot{x}_2$  :

$$m_1 \ddot{x}_1 = -b_1 (\dot{x}_1 - \dot{x}_2) - k_1 (x_1 - x_2) \quad (3)$$

$$m_2 \ddot{x}_2 = b_2 \dot{w} + k_2 w - b_1 (\dot{x}_1 - \dot{x}_2) - k_1 (x_2 - x_1) \quad (4)$$

Equations (3) and (4) is a second-order differential equations of a passive suspension system. Solving this systems of equations is difficult so we can use Matlab Simulink software. Solving the system and its verification will be done by three methods:

- write the equations in Matlab using Simulink library blocks;
- using the "transfer function";
- using the "state-space" model.

### 3. Graphic representation using Matlab library blocks

In Fig. 2 is represented the equation (3) and in Fig.3 to equation (3) is added the equation (4), now the system is completely. Solving was

performed using Simulink library blocks computing.

In order to analyze the behavior of the quarter car suspension system were used as input parameters the next values:

- $m_1 = 466.5 \text{ kg};$
- $m_2 = 49.8 \text{ kg};$
- $k_1 = 5700;$
- $k_2 = 135000;$
- $b_1 = 290;$
- $b_2 = 1400;$
- $k_d = 5.52;$
- $k_p = 10.0;$

corresponding to the above equations.

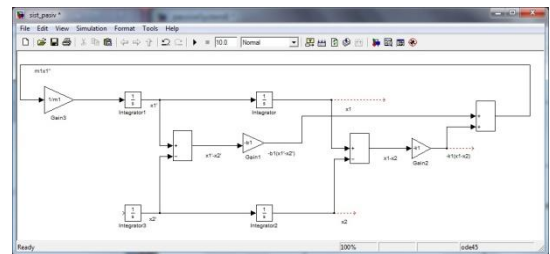


Figure 2 Matlab representation of the equation (3)

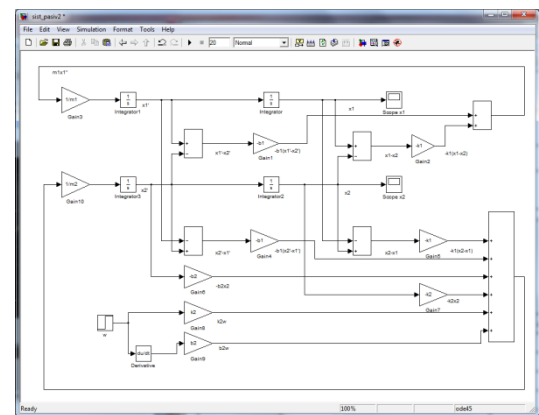


Figure 3 Matlab representation of the equation (3) and (4)

Given the input parameters that we run the program and the graphical representation of the system of equations, we can calculate the mass movements of the vehicle and its suspension. In Fig. 3 is presented car body displacement, it is noted that the excitation signal on mass m1 displacement typically occurs over a period of 15s after the oscillation amplitude is zero. In terms of comfort, passenger severe amplitudes persist during the 10s minimal except that the first oscillations are strongest.

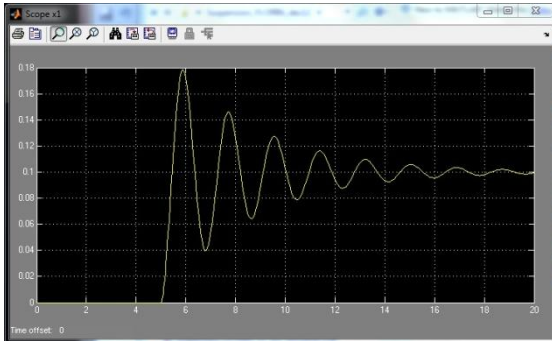


Figure 3 Car body displacement (unsprung mass)  $x_1$

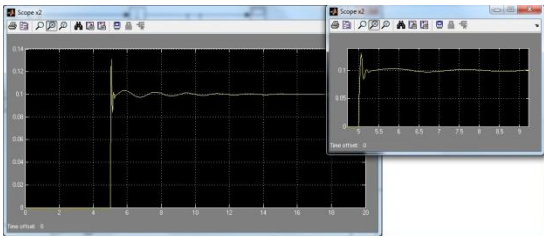


Figure 4 Suspension displacement (sprung mass)  $x_2$

#### 4. Graphic representation using Transfer function

To verify the results from Fig.3 and Fig. 4 we move on to the second method of calculation, using the "transfer function". Solving the system of equations (1) and (2) we will do, by using the Laplace transform, switching the original into a image function of a complex argument  $s$ .

The system of equations is:

$$m_1 x_1 s^2 + b_1 (x_1 - x_2) s + k_1 (x_1 - x_2) = 0 \quad (5)$$

$$m_2 x_2 s^2 + b_1 (x_2 - x_1) s + k_1 (x_2 - x_1) + b_2 (x_2 - w) s + k_2 (x_2 - w) = 0 \quad (6)$$

From the first equation (5) of the system is obtained:

$$x_2(s) = x_1(s) \frac{m_1 s^2 + b_1 s + k_1}{b_1 s + k_1} \quad (1)$$

If you add the equations (5) and (6) of the system and  $x_2(s)$  will be replaced, will obtains:

$$m_1 s^2 x_1(s) + x_1(s) \frac{(m_1 s^2 + b_1 s + k_1)(m_2 s^2 + b_2 s + k_2)}{b_1 s + k_1} = (2) = w(s)(b_2 s + k_2)$$

The transfer functions for the displacement of  $m_1$  and  $m_2$  are:

$$H_1(s) = \frac{x_1(s)}{w(s)} \quad (3)$$

$$H_2(s) = \frac{x_2(s)}{w(s)} \quad (4)$$

Using equations (8), (9) and (10) can be determined the transfer function  $H_1(s)$ :

$$H_1(s) = \frac{(b_2 s + k_2)(b_1 s + k_1)}{m_1 s^2 B_1 + B_2 B_3} \quad (5)$$

$$B_1 = (b_1 s + k_1)$$

$$B_2 = (m_1 s^2 + b_1 s + k_1)$$

$$B_3 = (m_2 s^2 + b_2 s + k_2)$$

If the variable  $s$  is separated, the transfer function becomes:

$$H_1(s) = \frac{b_1 b_2 s^2 + (k_1 b_2 + b_1 k_2) s + k_1 k_2}{m_1 m_2 s^4 + A_1 s^3 + A_2 s^2 + A_3 s + k_1 k_2} \quad (12)$$

$$A_1 = (m_1 b_1 + m_1 b_2 + m_2 b_1)$$

$$A_2 = (m_1 k_1 + m_1 k_2 + m_2 k_1 + b_1 b_2)$$

$$A_3 = (b_1 k_2 + k_1 b_2)$$

Equation (12) we write in Matlab using block "transfer function", its representation is in Figure 5.

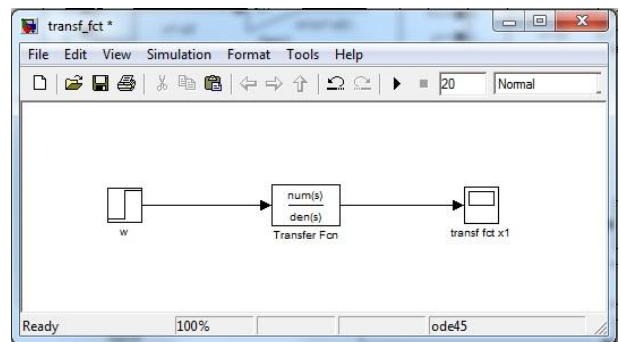


Figure 5 Transfer function

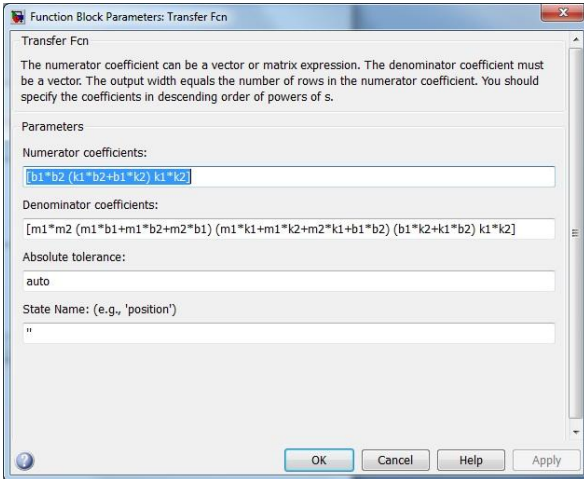


Figure 6 Transfer function parameters

After entering the parameters in this function and initial parameters of the suspension system described in Fig. 1 and run the program we get  $x_1$  moving vehicle mass as it shown in Fig. 7. We note that the displacement calculated by the "transfer function" is identical to that in Fig. 3, is the initial displacement calculated by solving the system of equations with Matlab block diagram.

Using the equation (10) we can calculated the suspension mass displacement in the same way as we calculated the car mass displacement.

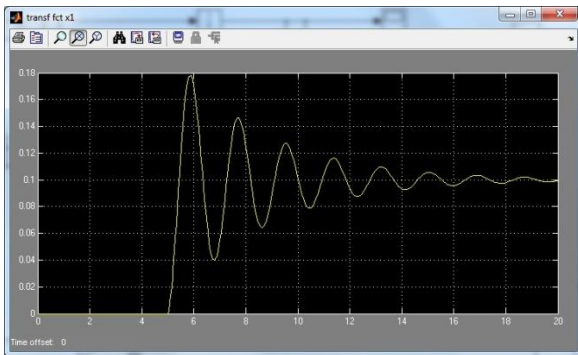


Figure 7 Car body displacement (unsprung mass)  $x_1$  calculated with transfer function

### 5. Graphic representation using State-space model

Another method of modeling the suspension system whit Matlab software is using the general form of the State-space model:

$$\begin{aligned} \dot{X}(t) &= A(t)X(t) + B(t)U(t) \\ Y(t) &= C(t)X(t) + D(t)U(t) \end{aligned} \quad (13)$$

where,  $X(t)$  is the "state vector",  $Y(t)$  - is the "output vector",  $U(t)$  - is the "input (or control) vector",  $A(t)$  - is the "state (or system) matrix",  $B(t)$  - is the "input matrix",  $C(t)$  - is the "output matrix",  $D(t)$  - is the "direct transmission matrix".

The matrix  $X(t)$  contains the following variables:

$$X(t) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_1 \\ x_2 \end{bmatrix} \quad (14)$$

from equations (5), (6), (13) and (14) "state space" matrixes of the system will be:

$$\begin{aligned} \dot{X}(t) &= \begin{bmatrix} -\frac{b_1}{m_1} & \frac{b_1}{m_1} & -\frac{k_1}{m_1} & \frac{k_1}{m_1} \\ \frac{b_1}{m_2} & -\frac{b_1+b_2}{m_2} & \frac{k_1}{m_2} & -\frac{k_1+k_2}{m_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} X(t) + \\ &+ \begin{bmatrix} 0 & 0 \\ \frac{b_2}{m_2} & \frac{k_2}{m_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{w} \\ w \end{bmatrix} \\ &= \begin{bmatrix} x_1 \\ x_2 \\ x_1 - x_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} X(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{w} \\ w \end{bmatrix} \end{aligned} \quad (15)$$

Matlab representation of equations (15) are made using a predefined block where are introduced the data from matrices.

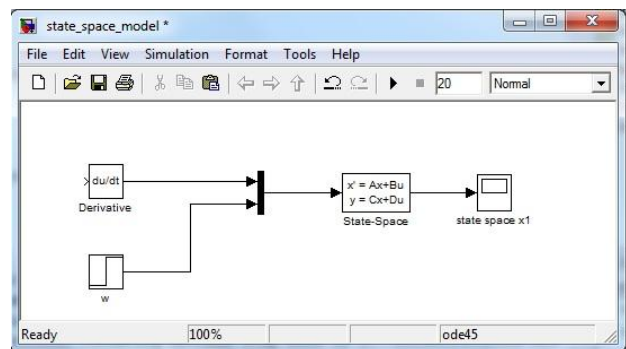


Figure 8 State-space model

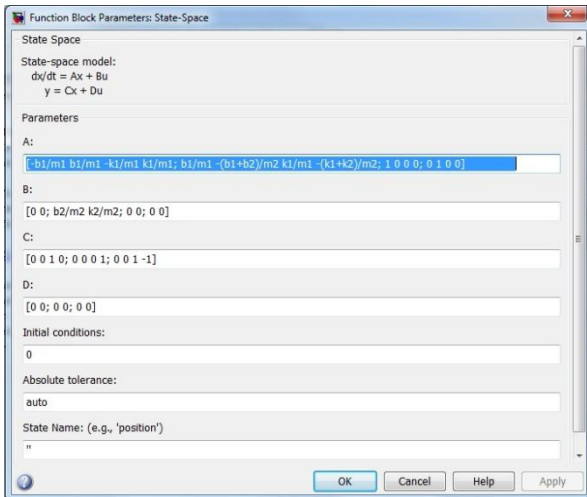


Figure 9 State-space model parameters

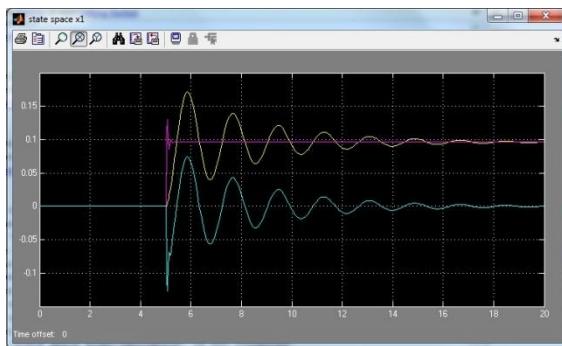


Figure 10 Car body displacement (unsprung mass)  $x_1$  calculated with State-space model

In Fig. 10 the yellow line from the graph above, represent the vehicle mass displacement  $x_1$ , the displacement is identical to those shown in Fig. 3 and Fig. 7. The purple line is the suspension mass displacement  $x_2$ , this is similar to the one shown in Fig. 4. The blue line represent the difference  $x_1-x_2$ .

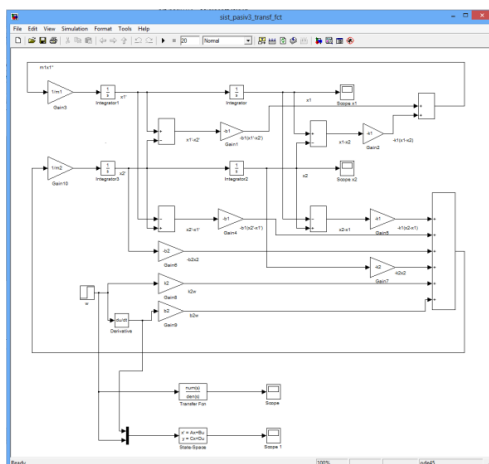


Figure 11 Matlab representation of the entire system

W is the excitation signal representing the road surface, in our case is a step type signal, its parameters are represented in Fig. 12.

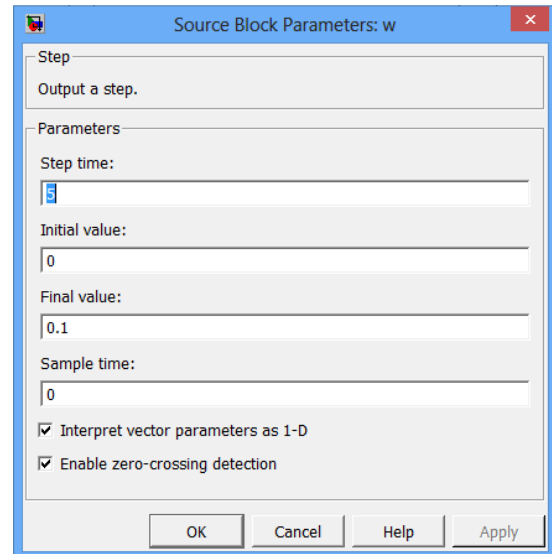


Figure 12 Step signal parameters

## 6. Conclusions

In this paper we simulated a passive suspension system, a system that is most commonly used.

To verify the accuracy of modeling were used State Space and Transfer Function.

Results obtained, using the three methods with the same parameters of the suspension system, are identical.

Was used a step type signal for a broad application of the suspension system. This signal can be modified, for example, a sinusoidal or a required signal.

The parameters of a passive suspension system are generally fixed, being chosen to achieve a certain level of compromise between road holding, load carrying and comfort.

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