# RESEARCH ON STRESS AND DEFORMATION STATE IN PLASTICAL BENDING WITH STRETCHING

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**Abstract** *The paper presents a theoretical solution of the stress and deformation state in plastically bending with stretching. It was established calculus relations of the neutral layer relative displacement coefficient, bending with stretching relative moment and elastic recovery size after elastic stress unloading are presented. It also is presented graphics showing the influence of relative radius to these sizes.*

**Keywords**: *stress, deformation, neutral stratus, bending with stretching, relative moment, elastic recovery.*

## 1. Introduction.

The working by bending is a plastically flexion of metal work-pieces around a edge thus the metal layer sited to curvature external are stretching in longitudinal direction producing metal elongation, and metal layer from curvature center are compressing, producing metal shorting [1].

Between the stretched and compressed layers is being neutral layer *MN* with the $\rho_n$ radius (fig.1).

The radius $\rho_n$ value where the tangential direction stresses are null and is not produced the deformation in tangential direction is determined by the following relation [2]:

$$\rho_n = \sqrt{R\, r} \tag{1}$$

or

$$\rho_n = r + x_0\, s \tag{2}$$

where $x_0$ can be determined with the relation:

$$x_0 = \sqrt{\left(\frac{r}{s}\right)^2 + \frac{r}{s}} - \frac{r}{s} \tag{3}$$

where: *R* is external bending radius; *r*- internal bending radius; $x_0$ - displacement coefficient of neutral bending layer.

The neutral layer with the radius $\rho_n$ is displaced to the curvature center compared with weight center, what is placed on the median arc with radius $\rho_m$.

In a normal section of bended wok-piece, working the stress in tangential direction, what linear are growing to external and internal metal layers from $\sigma_c$ to $\sigma_{real}$ values.

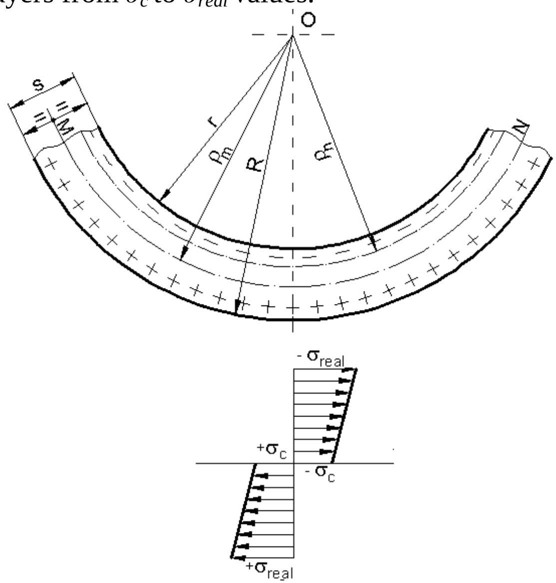


**Figure 1** - *Radial distribution of tangential stress in plastically bending with cold-hardening.*

Practically, it is considered that the neutral layer position $\rho_n$ coincides with the median layer with $\rho_m$ radius when the relative radius bending has the value $\rho_m/s > 5$ [3].

At relative radius bending $\frac{\rho_m}{s} > 200$ the tangential stress distribution can be determinate by relation [3].

$$\sigma_{real} = \sigma_c + E_1 \frac{y}{\rho_m} \tag{4}$$

where: $\sigma_c$ is metal flowing limit at stretching; $E_1$ - metal cold hardening modulus; $y$ - current position layer compared with neutral layer.

Metal cold hardening modulus can be established by relation [2].

$$E_1 = \frac{2{,}1\sigma_c}{\varepsilon_g} \tag{5}$$

where: $\varepsilon_g$ is metal relative strangling elongation.

Bending internal stress moment of a metal strip with the thickness $s$ and the wide equal with the unit can be determined by relation:

$$M = \left(\frac{S}{W} + \frac{E_1}{2\sigma_c}\frac{s}{\rho_m}\right) W\sigma_c \tag{6}$$

or

$$M = \left(K_1 + K_2 \frac{s}{\rho_m}\right) W\sigma_c \tag{7}$$

Where $S$ is static moment of section in relation with symmetrical axis; W- resistance modulus of section; $K_1 = \frac{S}{W}$ - coefficient defined by transversal section form; $K_2 = \frac{E_1}{2\sigma_c}$ - metal cold hardening coefficient.

If the bending relative moment is defined by relation:

$$m = \frac{M}{W\sigma_c} \tag{8}$$

The relation (7) can be writing:

$$m = K_1 + K_2 \frac{s}{\rho_m} \tag{9}$$

After plastically bending is canceled external plastically bending moment, the metal stripe being under internal moment action is elastic reloaded, producing elastic recovery, analytical established by relation [3]:

$$\Delta\alpha = 2m \frac{\sigma_c}{E}\frac{\rho_m}{s}\alpha_0 \tag{10}$$

where: $\Delta\alpha$ is elastic recovery angle; $E$- elastically metal modulus; $\alpha_0$ - metal strip bending angle.

The median layer radius value after elastic recovery $\rho_{mr}$, can be easy obtained equalizing the neutral layer length after bending with the neutral stratus length after elastic recovery, and with approximation $\rho_n = \rho_m$, it is obtaining:

$$\rho_{mr} = \frac{\rho_m}{1 - 2m\frac{\sigma_c}{E}\frac{\rho_m}{s}} \tag{11}$$

The relation (10) analyze highlights that elastic recovery is linear influenced by relative plastic bending moment, physical and mechanical metal characteristics, median relative bending radius, and bending angle.

It is apparently that elastic recovery of a piece influences the dimensional and form precision of a piece processed by bending.

**2. Neutral layer position determination and banding with stretching moment calculus with a tension $\sigma_t \le \sigma_c$.**

Many pieces categories with high dimensional and form precision are processing by bending with stretching, because after deformation load canceling the elastic recovery is lower.

The pieces processing by plastically bending with stretching is a process compound by plastically bending and axial stretching of metal stripe.

It is considered a metal stripe with the thickness $s$ and the wide equal with the unit and is loaded at stretching with the force $F$ what determines in a section a tension $\sigma_t \le \sigma_c$, then is bending with a plastically deformation external moment $M$.

The stress generated in stripe section for each load is presented in figure 2.

At plastically bending with stretching, successively loading with the axial force $F$, and then with the moment $M$, by overlapping effects in bending section zone will are obtained a stress state where in stretching layers will are contained higher stress with $\sigma_t$

value, and in these compressed reduced with $\sigma_t$ value.

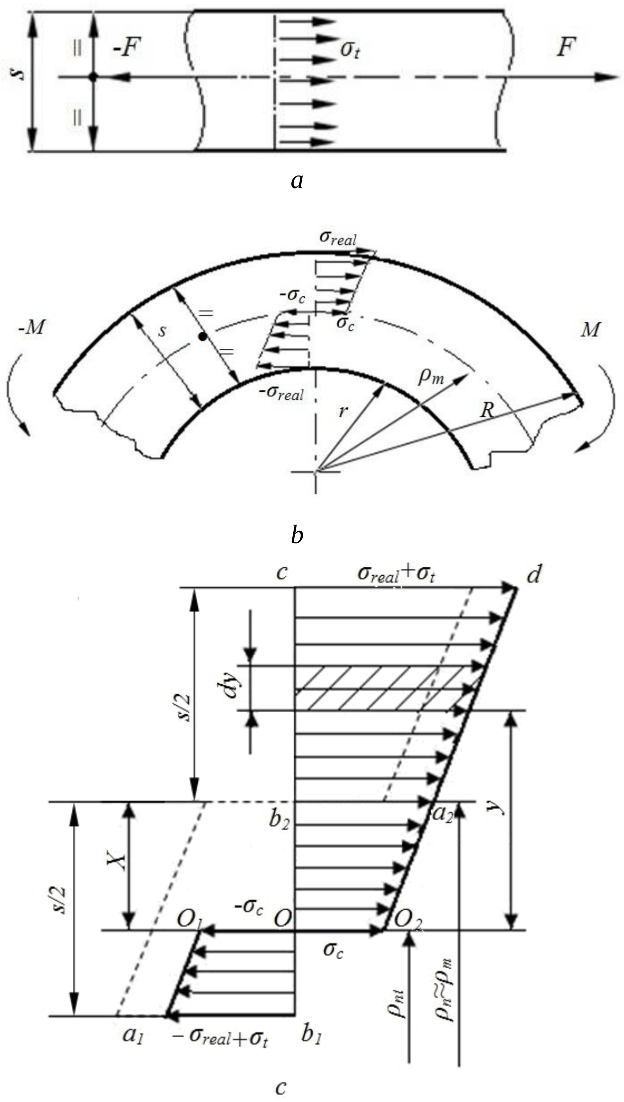


**Figure 2**. *Stress state in plastically bending with a tensile $\sigma_t \leq \sigma_c$ stress distribution at tensile with axial force F; b- stress distribution in plastically bending; c-stress distribution in plastically bending with a stretching obtained by overlapping effects.*

This resulted stress state of the bending with stretching is presenting in figure 3.

By applying tensile force *F*, after bending neutral layer with radius $\rho_n$ is moving to the center of curvature with distance *X*, to the position $\rho_{nt}$.

Analyzing stress distribution, in this case it can be considered that the stress determine trapezium surface $OO_1a_1b_1$ balances the stress determine trapezium surface $OO_2a_2b_2$. The stress on two trapezes balance stress produced by plastically bending moment *M*.

The stress determine surface $a_2b_2cd$ balance the stress produced by stretching force *F*.

The neutral layer displacing X value can be established with relation that expresses the sum condition of all forces acting on a normal section of the strip is equal to zero, thus:

$$\sum F_i = F_{oo_2dc} - F_{oo_1a_1b_1} - F = 0 \qquad (11)$$

or:

$$\int_0^{\frac{s}{2}+X} \left( \sigma_c + \frac{E_1}{\rho_n} y \right) dy + \int_0^{\frac{s}{2}-X} \left( \sigma_c + \frac{E_1}{\rho_n} y \right) dy - \sigma_t s = 0 \qquad (12)$$

Integrating, making necessary changes in equation (12) and the approximation $\rho_n = \rho_m$ is obtained neutral layer displacement calculus by relation:

$$X = \frac{\sigma_t s}{2\sigma_c + \dfrac{E_1}{\rho_m} s} \qquad (13)$$

If is expressed dimensionless *X* size, reporting at thickness *s* and note metal hardening coefficient $K_2 = E_1/2\sigma_c$, the relative displacement *x* of the size neutral layer will be:

$$x = \frac{X}{s} = \frac{\sigma_t}{\sigma_c} \frac{1}{2\left(1 + K_2 \dfrac{s}{\rho_m}\right)} \qquad (14)$$

The relative displacement of neutral layer, function by $\sigma_t/\sigma_c$ and relative thickness $s/\rho_m$ of a carbon steel strip with carbon content under *0,1%*, $\sigma_c = 210\ MPa$, and $\varepsilon_q = 34\%$ is presented in figure 3.

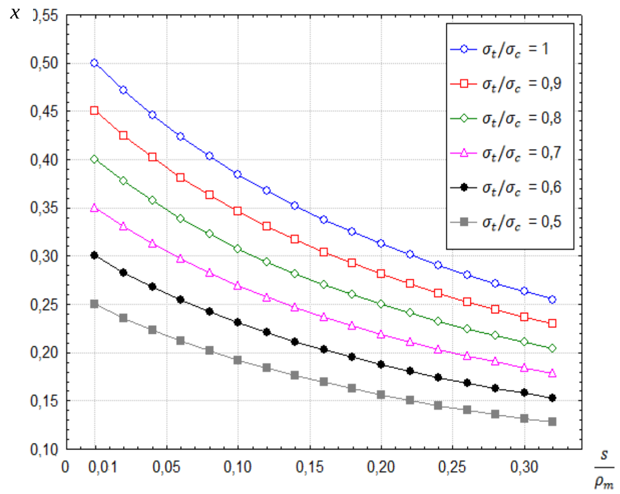


**Figure 3**. *Relative displacement of neutral layer x function by $\sigma_t/\sigma_c$ and $s/\rho_m$*

It is observe that the relative displacement $x$ increases with ratio increasing $\sigma_t/\sigma_c$ and decreases with increasing of relative bending with stretching $s/\rho_m$ .

**3. Internal plastic bending with stretching moment calculus ($\sigma_t \leq \sigma_c$).**

The internal plastic bending with stretching moment calculus $M$, can be easy determined using condition of energetically.

Thus, it is writing equality between mechanical work of external deformation moment with mechanical work of inner tension moment, it is obtaining:

$$M = \int_0^{\frac{s}{2}+X} \left(\sigma_c + \frac{E_1}{\rho_n}\right) y\, dy + \int_0^{\frac{s}{2}-X} \left(\sigma_c + \frac{E_1}{\rho_n}\right) y\, dy - \sigma_t sX$$

Integrating and approximating $\rho_n \approx \rho_m$ is achieved:

$$M = \sigma_c\left(\frac{s^2}{4} + X^2\right) + \frac{E_1}{\rho_m}\left(\frac{s^3}{12} + sX^2\right) - \sigma_t sX \quad (15)$$

Taking account by relation (13), moment value M defined by relation (15) becomes:

$$M = \sigma_c\left(\frac{s^2}{4} - X^2\right) + \frac{E_1}{\rho_m}\frac{s^3}{12} \qquad (16)$$

If for a band section $1$x$s$ note $W=s^2/6$, $K_1=S/W$ and $K_2=E_1/2\sigma_c$, is obtain:

$$M = W\sigma_c\left[K_1\left(1-4x^2\right) + K_2\frac{s}{\rho_m}\right] \qquad (17)$$

If is noted $m=M/W\sigma_c$ the relative plastically bending with stretching moment, the relation (17) obtains a dimensionless form, thus:

$$m = K_1\left(1-4x^2\right) + K_2\frac{s}{\rho_m} \qquad (18)$$

It is observed if $x=0$ ($\sigma_t=0$), the relation (18) is transforming in relation (7), defined at bending without stretching.

The variation relative bending moment with and without stretching, function by relative thickness, is shown in figure 4.

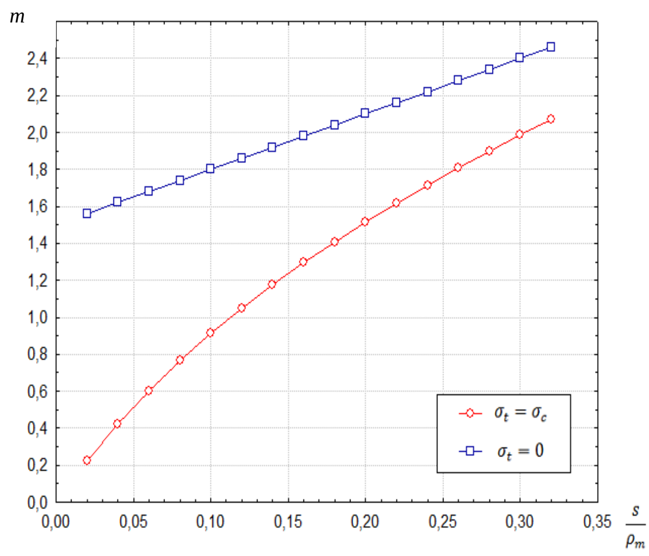


**Figure 4**. *Relative bending moment with and without stretching m function by relative thickness s/$\rho_m$*

The figure shows that the relative bending with stretching moment increases with increasing relative thickness. Also notice that when bending without stretching is relatively higher than the bending with stretching.

**4. Determination of elastic recovery size in plastically bending with stretching ($\sigma_t \leq \sigma_c$).**

After removal external loads, strip metal deformed by bending with stretching changes its sizes. This because the action of presence elastic deformations at plastic deformation law.

Cancellation moment external determines increases radius and decreasing bending angle, and stretching force cancellation produces shorter length strip and corresponding lower of bending angle.

Neutral layer radius value after removal of external bending moment is determined from the relation [1]:

$$\frac{1}{\rho_{ntr}} = \frac{1}{\rho_{nt}} - \frac{M}{EI} \qquad (19)$$

where:

$$\rho_{ntr} = \rho_{mr} - X \qquad (20)$$

$$\rho_{nt} = \rho_m - X \qquad (21)$$

$\rho_{ntr}$ is neutral layer radius after elastic recovery; $\rho_{nt}$ - neutral stratum radius in bending with stretching; *E*- elastic longitudinal modulus; *I*- axial inertial moment of section.

If in relation (19) is replaced $M=\sigma_c Wm$ and $I=s^3/12$ is obtained:

$$\rho_{ntr} = \frac{\rho_{nt}}{1 - 2m\dfrac{\sigma_c}{E}\dfrac{\rho_{nt}}{s}} \tag{22}$$

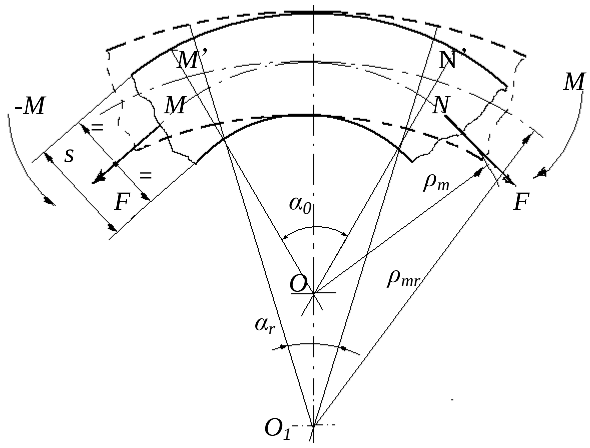


**Figure 5**. *Elastic recovery in banding with stretching*

Considering that the bending with stretching the neutral stratum radius is great, result that moving of neutral layer is very small relative to neutral bending radius; to simplify the relation (22) can be used midrange $\rho_{mr}$ and $\rho_m$.

Size elastic recovery $\Delta\alpha$ after cancellation the external loads when using overlapping effects of bending moment *M* and strength force *F*, can be written as:

$$\Delta\alpha = \Delta\alpha_{\hat{i}} + \Delta\alpha_t \tag{23}$$

where

$$\Delta\alpha = \alpha_0 - \alpha_r \tag{24}$$

$\alpha_0$ is bending with stretching angle; $\alpha_r$-angle after elastic recovery because the cancellation of *F* and *M*;

$$\Delta\alpha_{\hat{i}} = \alpha_0 - \alpha_{r\hat{i}} \tag{25}$$

$\Delta\alpha_{\hat{i}}$- elastic recovery angle because cancellation of external moment *M*; $\alpha_{r\hat{i}}$- angle after cancellation of *M;*

$$\Delta\alpha_t = \alpha_{r\hat{i}} - \alpha_{rt} \tag{26}$$

$\Delta\alpha_t$ is elastic recovery angle because cancellations of strength force *F;*

$\alpha_{rt}$ – angle after cancellation of *F;*

The $\Delta\alpha_{\hat{i}}$ size can be determined from equalizing length neutral layer bending with stretching radius $\rho_{nt}$, with length neutral layer after canceling the bending moment $\rho_{ntr}$ radius, thus:

$$\rho_{nt}\alpha_0 = \rho_{ntr}\alpha_{r\hat{i}} \tag{27}$$

In relation (27) after replacing $\rho_{ntr}$ with the value defined by relation (22), resulted:

$$\Delta\alpha_{\hat{i}} = 2m\frac{\sigma_c}{E}\frac{\rho_{nt}}{s}\alpha_0 \tag{28}$$

The $\Delta\alpha_t$ size is determined by equalizing shortening equalization neutral layer with arc length subtended by angle $\Delta\alpha_t$:

$$\Delta l_t = \Delta\alpha_t\ \rho_{ntr} \tag{29}$$

$$\Delta l = \frac{\sigma_t}{E} l \tag{30}$$

$$l = \rho_{nt}\alpha_0 \tag{31}$$

From these relations result:

$$\Delta\alpha_t = \frac{\sigma_t}{E}\frac{\rho_{nt}}{\rho_{ntr}}\alpha_0 \tag{32}$$

By replacing relations (28) and (32) in relation (23) is obtained:

$$\Delta\alpha = \left(2m\frac{\sigma_c}{E}\frac{\rho_{nt}}{s} + \frac{\sigma_t}{E}\frac{\rho_{nt}}{\rho_{ntr}}\right)\alpha_0 \tag{33}$$

If we consider that the displacement of neutral layer after elastic recovery is very small compared to the bending radius, at high levels it can be considered $\rho_{nt} \approx \rho_{ntr} \approx \rho_m$ and relation (33) become:

$$\Delta\alpha = \left(2m\frac{\sigma_c}{E}\frac{\rho_m}{s} + \frac{\sigma_t}{E}\right)\alpha_0 \tag{34}$$

On figure 6 is presented elastic recovery variation angle function by relative bending radius at 90°, bending with the stretching $\sigma_t = \sigma_c$ and $\sigma_t = 0$.

Graphic analysis shows that at plastic bending with stretching the elastic recovery angle is much smaller compared with the simple plastic bending.

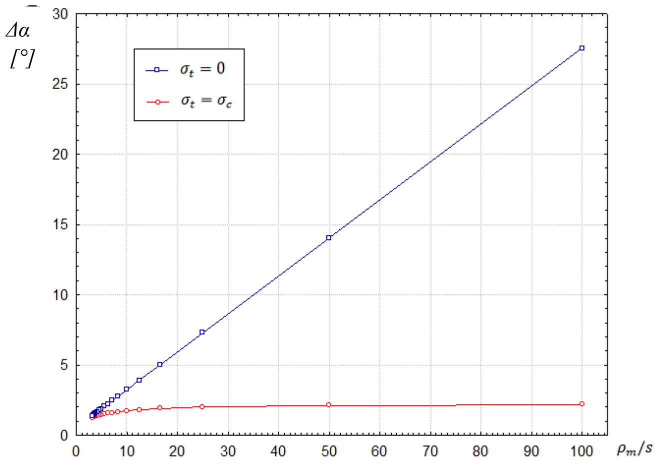


**Figure 6.** *Variation plastic recovery angle in bending at 90° function by $\rho_m/s$ and stretching with $\sigma_t = \sigma_c$ and $\sigma_t = 0$.*

Elastic recovery reduction efficiency at plastically bending with stretching increases comparatively simple plastic bending when relative radius bending has higher values.

## 5. Conclusions

The working by plastically bending with stretching is a process what can assure dimensional and form with high precision at pieces obtained by bending with great relative radius comparative with the simple plastic bending.

By stretching take place a pronounced displacement of neutral layer to compressed layers. This aspect contributes at diminished of internal plastically bending.

The results obtained in mathematical modeling of elastic recovery lead us to the conclusion that in bending with stretching it decreases considerably compared with that obtained from simple plastic bending.

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