CALCULATION OF THE PRESSURE OF THE LUBRIFICATION FILM IN THE VARIABLE STRESSED SURFACE

Valeriu Certan

Technical University of Moldova

Abstract: *This paper presents a simpler form of deriving the pressures' equation taking as starting point the Reynolds equation for hydrodynamic bearing and the general movement solution for thick-walled cylinder loaded with proportional pressure sinθ.*

Key words: *Sliding bearing, hydrodynamic lubrication, pressure, lubricant spin, lubricant film thickness.*

1. Introduction

With the appearance of new materials that have satisfactory resistance the use of sliding bearings in various branches of engineering broadens. Short bearings which length is part of the diameter have a widespread use. The use of short bearings has as aim the reduction of overall weight and size dimensions.

For various reasons the functioning of short bearings was less researched. One of the most known papers is Riebe A., Falţ E., Frene J., Nicolas D., Degueurce B [9,3], etc.

Despite the widespread use of short bearings, their calculation is not yet satisfactory.

It is known that the main factor which determines the bearing capacity and reliability of the surface is the layer of lubricant which separates the sliding surfaces. However, of further importance remains the issue about the influence of the *length of the bearing* on the thickness of the lubricant layer, *variable* cyclic load etc.

The theory of lubrication for long bearings gives satisfactory results for the calculation of long bearing because the length is an auxiliary factor which can be approximated with a parabolic or another correction coefficient. However, when the length of the bearing becomes small measure, the theory of long bearings is less used. It may be used only for qualitative analysis.

Currently the research in this area is focused on the theory of finite length bearing lubrication.

In this area it is worth mentioning the following papers Hans Reissner [8], Muskat and Morgan [4, 2, 5].

H. Reissner has analysed the issue of constant load bearing for the condition of existence of a continuous lubrication layer in empty space. However this condition is fulfilled only for eccentricity less than 0.5 as recommended by Zommerfelid [6]. The results of H. Reissner are far from a final solution yet.

This paper attempts to apply the schematic flow model to determine the pressure in the continuous lubricant layer in empty space for the finite length bearing under the action of variable in size load without taking into account the influence of the rotational movement of the slide.

2. Basic equation and the boundary conditions of the problem

The equation for the distribution of pressure in the lubricant layer has been derived by Reyolds, [1].

For the general case this equation has the following form

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial z}\right)=$$
$$=\eta\left[\left(U_0+3U_1\right)\frac{\partial h}{\partial x}+2V\right]$$

where

x – is the coordinate on the sliding surface in the direction of the relative velocity;

z – is the coordinate on the sliding surface in the direction perpendicular to the direction of the relative movement;

y – is the coordinate perpendicular to the sliding surface;

U_0 – is the velocity for $y=0$;

U_1 – is the velocity for $y=h$,

$$V=\frac{dh}{dt}\cdot$$

For the case under consideration which assumes that the slide is not rotating

$$U_0=0$$

and

$$U_1=0.$$

Consequently the O. Reinolds' equation takes the following form

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=12\eta\frac{dh}{dt}\cdot$$

The thickness of the lubricant film has the following form

$$h=\delta-e\cos\varphi\ ,$$

where - $\delta=R-r$; e - is the distance between centres (fig.1)

$$e=OO_1.$$

In this case

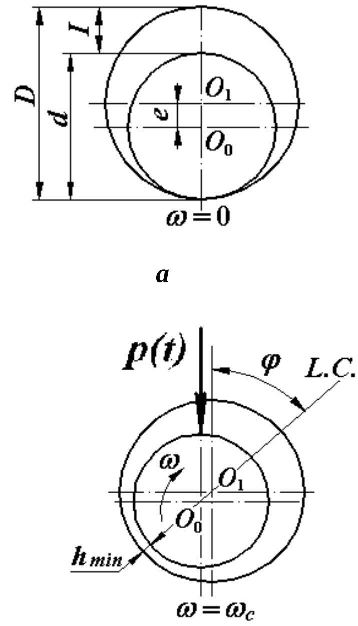
$$\frac{dh}{dt}=-\cos\varphi\frac{de}{dt}\ .$$

By replacing x with the product of radius r and angle φ, we obtain

$$\frac{\partial}{r\partial\varphi}\left(h^3\frac{\partial p}{r\partial\varphi}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)+12\eta\cos\varphi\frac{de}{dt}=0 \quad (1)$$

$$y=0$$

The origin of the coordinate system is taken at the surface of the front of the slide. The slide length is denoted by l. The surface boundary conditions for the case under consideration are: $p=0$ for $z=0$, $z=l$ and $\varphi=\pm\frac{\pi}{2}$.



b

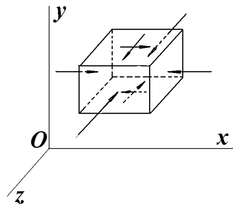
Figure 1

Basic deduction of O. Reinolds' equation

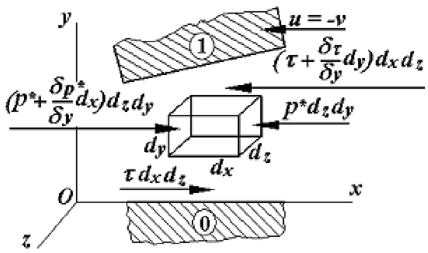
Let's assume that a liquid continuous medium of arbitrary viscosity is moving between two parallel plates A and B, which may become close to each other, remaining parallel. The liquid particles are moving under the action of difference in pressure and friction forces. The thickness of the liquid layer h will be considered as small measure compared to other dimensions. When considering the liquid film thickness as small measure, it can be considered that the pressure in the liquid layer is changing only along the plates, or in other words, in the direction of the x and z axes. With varying the pressure in the direction of thickness of layer the pressure will be neglected,

$$\frac{\partial p}{\partial y}=0.$$

Similarly (due to the thickness of the liquid layer as small measure and the parallelism of the plates) the change of the speed of the particles in the direction along the axis x and z can be neglected as small measures in comparison with the movement in the direction of the y axis.



a



b

Figure 2

Next we will consider the forces which stress the elementary (small) volume of a parallelepiped shape (fig. 2, *a*). On the sides perpendicular to the y axis, in the direction of the x axis the following friction forces work

$$-\eta\frac{\partial v}{\partial y}dxdz,$$

$$+\eta\left(\frac{\partial v}{\partial y}+\frac{\partial^2 v}{\partial y^2}dy\right)dxdz.$$

The friction forces on other facets can be neglected because the change of the speed on these facets is low.

The pressure forces act on the facets, perpendicular to x axis

$$pdydz,$$

$$-\left(p+\frac{\partial p}{\partial x}dx\right)dydz.$$

Figure 2, *a* presents the forces related to axes x and z. The low-order neglected forces in fig. 2, *a* are not presented.

Neglecting the inertial force of the particle, the equilibrium condition of forces leads to the following:

$$\frac{\partial p}{\partial x}=\eta\frac{\partial^2 v}{\partial y^2}.$$

After applying the double integration we obtain:

$$v=\frac{1}{2\eta}\frac{\partial p}{\partial x}y^2+ay+b.$$

The considered plates can only become closer to each other while remaining parallel to each other. Assuming that the hypothesis that the fluid sticks perfectly to the plate is true, we get that for boundary conditions $y=h$ and $y=0$

$$v=0.$$

Finally

$$v=\frac{1}{2\eta}\frac{\partial p}{\partial x}y(y-h).$$

The highest speed is obtained for

$$y=\frac{h}{2}.$$

In such case

$$v_{\max}=-\frac{1}{8\eta}\frac{\partial p}{\partial x}h^2.$$

The average speed in a section of height h, and width dz forms $\frac{2}{3}$ of the maximum value

$$v_{med}=-\frac{1}{12\eta}\frac{\partial p}{\partial x}h^2.$$

Let's assume that the top plate is moving downward so that it remains parallel to itself. At this moment in the considered layer variable pressures will appear. Consequently relationships will be established between the speed of the plate movement, $\frac{\partial h}{\partial t}$, the thickness of the liquid layer h and its internal pressure p. For this purpose the hypothesis of continuous medium will be applied. The amount of lubricant which flows from the elementary volume in the form of parallelepiped with the dimensions Δx, Δz, Δh, at the movement of the top plate downward at distance Δh will be equal to $\Delta x \times \Delta z \times \Delta h$ (fig. 3).

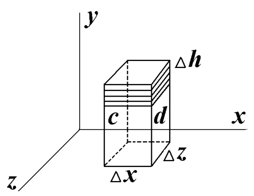


Figure 3

Let's calculate the amount of fluid that will drain through the side walls of the elementary parallelepiped.

The amount of fluid that will drain through the wall c which is also called the leakage will be:

$$Q_c = (v_{med})_x h_x \Delta z \Delta t = -\frac{1}{12\eta}\left(\frac{\partial p}{\partial x} h^3\right)_x \Delta z \Delta t$$

The amount of fluid that will drain through the side wall d of the elementary parallelepiped will be:

$$Q_d = -\frac{1}{12\eta}\left(\frac{\partial p}{\partial x} h^3\right)_{x+\Delta x} \Delta z \Delta t \,.$$

The difference between Q_d and Q_c will be:

$$Q_d - Q_c = -\begin{bmatrix} \frac{1}{12\eta}\left(\frac{\partial p}{\partial x} h^3\right)_{x+\Delta x} - \\ -\frac{1}{12\eta}\left(\frac{\partial p}{\partial x} h^3\right)_x \end{bmatrix} \Delta z \Delta t =$$

$$= \left[-\frac{1}{12\eta}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x} h^3\right) + s_1\right] \Delta x \Delta z \Delta t \,.$$

Similarly the quantity of liquid that will drain through the side walls of the elementary parallelepiped perpendicular to the axe z can be calculated:

$$\left[-\frac{1}{12\eta}\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial z} h^3\right) + s_2\right] \Delta x \Delta z \Delta t$$

Summing up the drained quantities, the *consumption*, when equalling to the volume $\Delta x \times \Delta z \times \Delta t$ and applying the limit leads to:

$$\frac{1}{12\eta}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x} h^3\right) + \frac{1}{12\eta}\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial z} h^3\right) = \frac{\partial h}{\partial t}.$$

This represents the equation for the distribution of pressure in lubricant layer obtained by Reinolds.

References

[1.] Reynolds O., Phil. Trans. Roy. Soc.177, 157, 1886.

[2.] Constantinescu, V.N. ş.a. Lagăre cu alone-care, Bucureşti Ed. Tehnica, 1980.

[3.] Pascovici M. Ş.s. Metoda generală pentru calculul geometric, hidrodinamic şi de rezistenţă pentru lagărele radiale cu alone-care hidrodinamică. TS46 Bucureşti, Inst. Politehnic, 1980.

[4.] Frene J, Nicolas D, Deguerce B, Berthe D, Godet M. Lubrication hidrodinamicque-paliers et debutees, Ed.Eyrolles, Paris 1990.

[5.] Hamrock B.J., Fundamentals of fluid film lubrification, R.R.Donneley & Sons Com-pany, New York 1994.

[6.] Muscat M., Morgan F. The' Theory of the Thick Film Lubrication of a Complete Journal Bearing of Finite Length // J. Appl. – 1938, Vol. 9. – p. 393-409.

[7.] Muscat M., Morgan F. The Theory of the Thick Film Lubrication of a Complete Journal Bearing of Finite Length with arbitrary of the lubrication source // J. Appl. Phus. – 1939 Vol. 10, nr.1, p. 46-61.

[8.] Sommerfeld A. Zur Theorie der Schmiermittel reiburg. // Zhscher. Techn. Phys. –nr.3, 4 . -1921

[9.] Hans Reisner, Zeits. f. angew. Math. u. Mech. 16, 275 , 1936.

[10.] Frene J., Nicolas D., Degueurce B.,Berthe D., Godet M. Lubrication hidrodinamique paliers et debutees Ed. Eurolles, Paris, 1990.