DEVICE AND METHOD FOR STUDYING THE DYNAMICS OF SYSTEMS SUBJECTED TO FRICTIONAL IMPACT. PART I: SLIDING FRICTION CASE

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Abstract: *The paper presents a method and test rig for the dynamic study of systems acted by percussion and sliding friction. The device consists in an assembly of three prismatic bodies in a sandwich pack. Keeping fix the inferior body and applying a normal percussion to the intermediate body, tangential percussions occur at the two contact surfaces producing the displacement on horizontal direction of upper bodies. Measuring the absolute displacements of the superior bodies by the use of non-contact sensors, the friction forces arising after impact can thus be found. The non-contact sensors are helpful not only in linear displacements, but, in the case when the upper body is a cylinder contacting the intermediary body along a generatrix, rolling occurs accompanying sliding. The method cannot be applied for this particular situation.*

Keywords: *system dynamics, percussions, Coulombian friction*

1. Introduction

The impact phenomenon is widespread in applied engineering. As a general characteristic, the rapid variation of kinematical parameters can be mentioned, [1]. The theoretical approaches of the phenomenon are simpler or more complex, depending on the assumption regarding the kinematical parameters variation: instantaneous or during a finite time period. As a consequence of instantaneous variation hypothesis, the impact forces cannot be estimated. With the supposition of parameters variation during finite time, the impact forces present a continuous variation during impact.

One of the simplest models studying the impact phenomenon considers that between the contacting bodies only normal interactions take place. Timoshenko, [2], gives the model for the impact of two perfect elastic spheres. Lankarani, [3], and Flores, [4], consider the model for the impact with internal damping of two spheres. The internal friction influences the system's dynamics by introducing the hysteresis loop. A parameter that quantitatively estimates the lost work due to internal friction is the coefficient of restitution. Another time, Lankarani, [5], proposes a model of impact with plastic deformations.

The hypothesis of plastic deformations is most truthful as Johnsosn, [6], stated. Johnson shows that a ball is impacting a plane steel surface with a velocity $v \geq 0.14 m/s$, intrinsically produces plastic strains. For the case when the relative motion between the contacting surfaces is a plane one and Coulomb friction is present between the bodies, Routh, [7], offered an extremely expedite graphical method for dynamical study of the system. Wang and Mason, [8], applied the method on a robot, for the study of prehensile force. Special, intricate cases arise when disjunction between static friction and dynamic friction is made, [9].

Theoretically, the friction force can take any value between two extreme values which depend on the static coefficient of friction μ_s and the normal loading force. At the instant when between the contacting points relative motion happens, the friction force has an unique value decided by normal loading force and by the dynamic friction coefficient, μ_d.

In order to emphasize the complexity of dynamic study for a system under impact with friction, the well-known example due to Kane is referred to: studying the impact of a double pendulum and a horizontal rough plane, Kane shows that, by applying for the coefficient of restitution the kinematical definition, the paradoxical situation is encountered that the kinetic energy after impact is greater than the initial kinetic energy.

The above mentioned considerations prove that the results of a theoretical model depend strongly on the adopted hypothesis and that, when possible, the study should preferably be made directly on a physical device.

2 Dynamical analysis using the proposed method

The study of dynamical behavior of systems under impact loads, with emphasize on the presence of tangential impact forces, is intended to be made on the device presented in Fig. 1. The device consists in a rigid frame, 1, to which a hammer, 2, is jointed. The active surface of the hammer is a bearing ball, with a $40mm$ diameter.

The ball is assembled to the hammer body by means of a conical reaming and fixed in three welding points so that the ball characteristics in the impact zone are not affected. A plate, 3, is firmly attached to the device base. Two prismatic parts, 4 and 5, are placed onto this plate. In the resting situation, the hammer must be tangent in the center of the vertical face of the lower mobile prism. Test rig operation assumes the rotation of the hammer, around an axis on the fix frame, with an angular magnitude ψ_{max}. The hammer is set free. At the moment the hammer reaches the vertical position, collides the inferior mobile prism. Due to impact, the lower prism takes a horizontal motion and the upper prism 5, depending on the value of coefficient of friction between the mobile prisms, will move at once with the inferior prism or, will present relative motion both with respect to the inferior prism and to the firm prism. In order to measure the friction force after impact, two aluminum rulers, 6, were placed on the two mobile prisms, to materialize the reflecting surfaces for the waves emitted by the non-contact ultrasound sensors, 7.

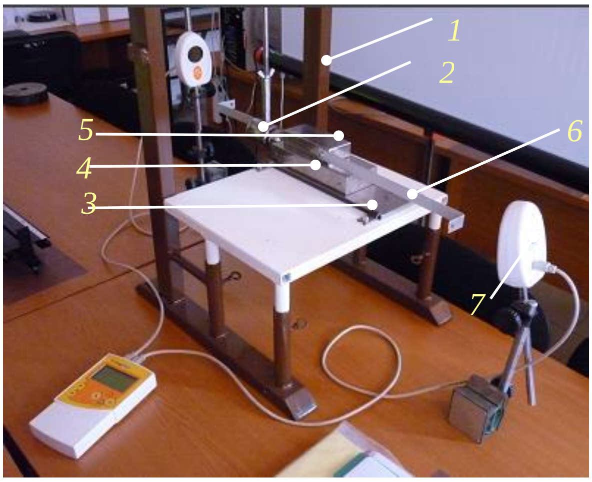


Figure 1: *Laboratory test rig*

The restitution coefficient for the impact between the hammer and the inferior prism is a parameter required for the dynamic study of the system. According to Lankarani's model, [5], plastic deformations are anticipated.

The impact print modifies the geometry of the impact region. It is preferable to effectuate some launchings of the pendulum, with the inferior prism positioned in the same place, with the purpose that the radius of the impact zone to increase up to a value over which the contact pressure would not produce additional plastic deformations.

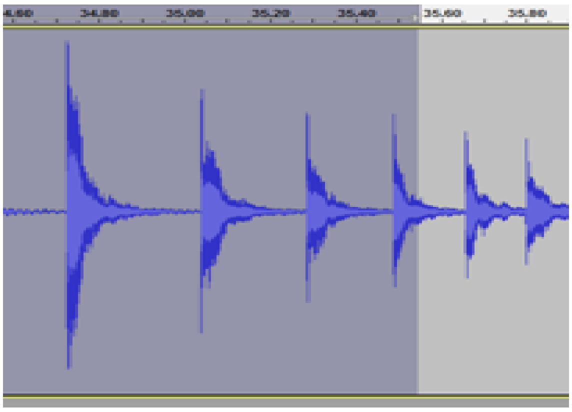
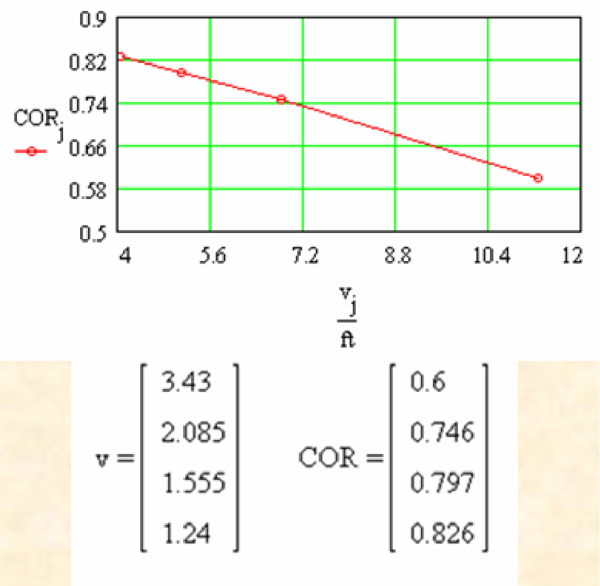


Figure 2: *The acoustic signal plot used for finding the coefficient of restitution*

The coefficient of restitution was found using the method proposed in [10]. Basically, the lower prism was slightly tilted and a ball identical to the one from the pendulum construction was set in free fall onto the prism, the acoustic signal generated by successive collisions being registered.



$$v = \begin{bmatrix} 3.43 \\ 2.085 \\ 1.555 \\ 1.24 \end{bmatrix} \quad COR = \begin{bmatrix} 0.6 \\ 0.746 \\ 0.797 \\ 0.826 \end{bmatrix}$$

Figure 3: *The restitution coefficient and is variation with the initial impact velocity*

The quasi-linear variation of the coefficient of restitution with the initial impact velocity, mentioned by Goldsmith, [11], is confirmed by the plot obtained, Figure 3. Comparing the graph obtained in Fig. 3 with the plots given by Goldsmith for different materials, it is observed that the plot correspond to an annealed steel.

3 Experimental results

During an experimental test, the two proximity sensors will simultaneously register the horizontal displacements of the two bodies. Assuming Coulombian friction, after impact, the friction force will be constant and consequently, the displacement will have a parabolic variation.

The signal provided by the sensor is differentiated twice with respect to time and after multiplying by body mass, the tangential force acting upon the body is thus obtained. Fig. 5 shows that a direct differentiation of the signal registered by the sensor leads to a result not easy to use.

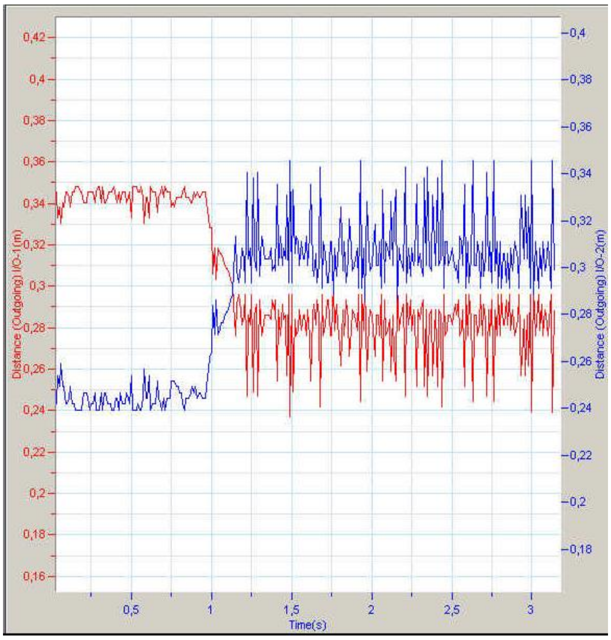


Figure 4: *Absolute positions of the two prisms, given by the proximity sensors.*

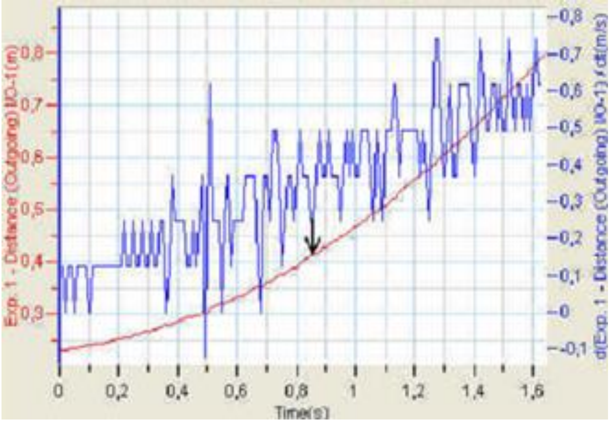


Figure 5: *The effect of direct derivation of the registered signal*

In order to avoid this aspect, the signal is first interpolated with a second degree polynomial, using the MULTILOGPRO software.

As mentioned before, an experimental test comprises simultaneous recording of the displacements of the two prisms and simultaneously pendulum's motion recording a movie using a video camera, with the objective lens axis coincident with the pendulum rotation axis.

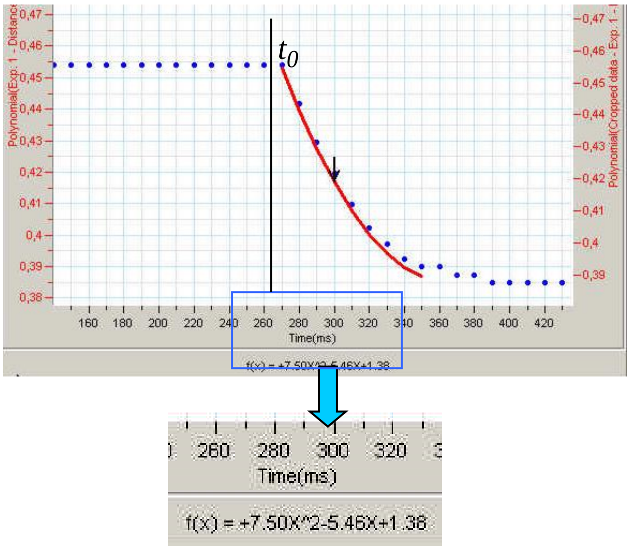


Figure 6: *Interpolated signal during the period of prism motion*

Subsequent to movie making, the velocities immediate after impact of the prisms, v''_1 and v''_2 are found and also, the maximum amplitudes of the hammer, before and after launching, ψ' and ψ'' respectively, are determined. In the laboratory, video recording of the hammer motion was made using a common video camera, oriented normal to the oscillating plane of the pendulum, Fig. 7, and the signal is presented in Fig. 9.

Figure 7: *Video recording of pendulum motion*

4 Internal percussions finding using the experimental results

The two prisms and the pendulum, instantly after impact are represented in Fig. 8.

The theorems of linear momentum and angular momentum with respect to point O, integrated between the initial and final impact instants for the two prisms and the hammer, 1, are written respectively:

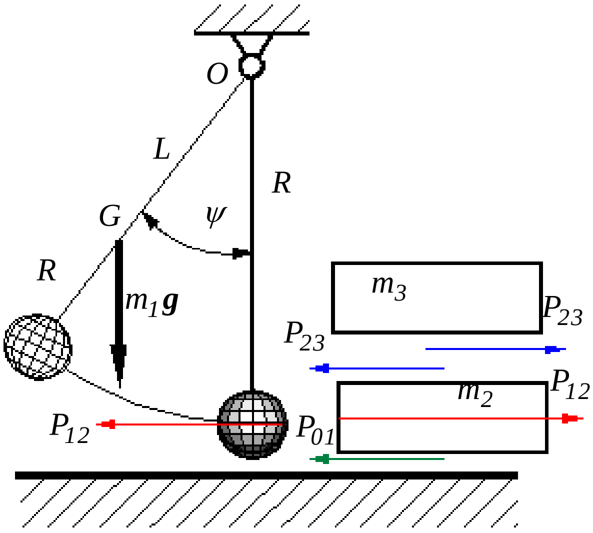


Figure 8: *Scheme for finding percussions*

$$\begin{aligned} & m_2(v''_2 - v'_2) = P_{23} \\ & m_1(v''_1 - v'_1) = P_{12} - P_{23} - P_{20} \\ & J_O(\omega'' - \omega') = -P_{21}R \end{aligned} \qquad (1)$$

where with $(\,'\,)$ and $(\,''\,)$ were denoted the parameters corresponding to the instants of initiation and final impact, respectively.

The system (1) has the unknowns: the normal percussion P_{12} occurring between the spherical hammer head and lower prism, and the tangential percussions, P_{20} and P_{23}, that happen at the contact surface between the fix prism and the mobile lower prism, and between the mobile prisms, respectively.

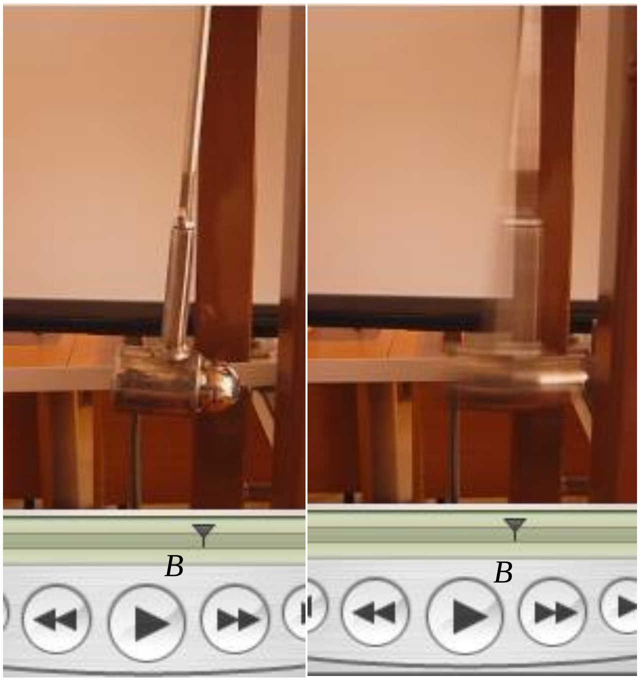
The inertial characteristics are known: the masses m_1, m_2 and m_3, the inertia momentum of the hammer with respect to oscillation axis, J_O, and also the position of the mass centre position G of the pendulum with respect to the oscillation axis, given by the distance L, and the lever arm of normal percussion P_{12} with respect to the oscillation axis of pendulum. The velocities of the two prisms before impact are zero.

$$v'_1 = 0, \; v'_2 = 0 \qquad (2)$$

The velocities after impact can be found by calculating the derivatives of the interpolation functions $s_2(t)$ and $s_3(t)$ at instant t_0 (end of collision) when they start to move. As it can be observed from Fig. 6., the t_0 instant can be found with a very good precision.
Thus:

$$v''_{2,3} = ds_{2,3}(t_0) \tag{3}$$

Finally, for finding the pendulum angular velocities, the movie with the pendulum rotation is used. In this movie, the angular amplitudes, ψ' and ψ'' of the pendulum can be precisely found. In the vicinity of the points of maximum amplitude, the angular velocity is almost zero and thus the position of maximum amplitude can be straightforwardly obtained.



a) b)

Figure 9: *Finding the pendulum maximum amplitude*

The button B for image playing, is moved left-right and the position of maximum angular amplitude is found when the course of rotation changes. This aspect can be observed in Fig. 9a. The pendulum instantly after impact is presented in Fig. 9b. From this figure, the pendulum's position cannot be accurately found, the movie being obtained at a speed of $30\ frms / sec$.

The energy theorem written for the pendulum between the instants of launching and the beginning of impact:

$$\frac{1}{2} J_O \omega'^2 = mgL(1 - \cos\psi') \tag{4}$$

allows for finding the angular velocity before impact. Analogous, the unknown angular velocity after impact is found using the relation:

$$\frac{1}{2} J_O \omega''^2 = mgL(1 - \cos\psi'') \tag{5}$$

Now we have all the data for finding the unknown percussions. To be noticed that the tangential percussions P_{23} and P_{01} are not a consequence of applying normal percussions and of Coulomb friction presence. This type of percussion is mentioned in the classification proposed by Wang and Mason, [8], as a distinct type of percussions.

Conclusions

The paper proposes a device and methodology for finding the percussions occurring in a dynamic system when one of the elements is subjected to impact. The major purpose of the work is to emphasize the existence of tangential percussions, a concept relatively newly introduced in literature. The presence of tangential percussion is the cause of motion arising (in the absence of a normal percussion) for a body from a system acted by impact. The dynamic equations are deduced and there are presented the methods for finding the required parameters for solving the dynamical equations system.

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