# REFLECTIONS ON THE MINIMUM THICKNESS OF THE LUBRICANT FILM AT SLIDER BEARING WITH CIRCULAR MOTION AND HYDRODYNAMIC LUBRICATION

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**Abstract:** *This paper presents the deduction of the equation for radial motion of points on the contact area based on Reynolds equation for short hydrodynamic bearing and the general motion solution for thick-walled cylinder with proportional load* $\sin\theta$.

*The obtained result appreciates the tensions and deformations that allow an optimal solution for the design of the bearing.*

**Key words:** *slider bearing, hydrodynamic lubrication, pressure, lubricant spin, tension, radial displacement, lubricant film thickness.*

## 1. Introduction

Slider bearings come in structures of mechanical systems as technical applications of basic cases to support shafts and axles that rotate in space on a fluid lubricant film, considering it is incompressible and has constant viscosity.

The use of these mechanical sub-assemblies for fixing shafts is widely used in turbine driven devices, submersible and sealed pumps which use as lubricant the low viscosity medium for lubrication and cooling: fuel, heat carriers, hydrocarbon condensate etc.

This poses special requirements when choosing the type of the bearing as well as the methods for the calculation and design.

One of the areas of interest at the moment is the theoretical and experimental deepening of the mechanics of contact in order to assess more accurately the bearing capacity and the sustainability of the elements which operate with sliding contact.

Most contacts encountered in technics work not only with normal load, but are also subject to tangential forces distributed on the contact area in the form of tangential tensions. The resultant of these tensions can be either a force inclined to the axis or directed by one of the axes, or a spin moment.

While tangential tensions induced by the load on the contact area are smaller in comparison with the normal pressure, they have a significant effect on the state of tension. Experimental tests have shown that durability of the contact is reduced considerably in the presence of tangential tensions. It is therefore necessary to study the state of tension produced in some of the bodies in contact by the mentioned distributions of tensions on the contact area such as normal and tangential.

The impossibility to solve analytically the integral of the pressures in the film spin, tensions and movements, is one of the reasons why so far no general solution has been found for the case when the pressure in the lubricant spin is considered as pressure. Therefore, solutions have been considered for specific cases.

For the linear contact, a primary solution is provided by E. M'Ewen [1], by using complex functions. Similar solutions were obtained by

H. Poritsky [2] by using Airy functions and by J. O. Smith and S. K. Liu [3] by using the real variables method.

The theoretical results were confirmed by experimental studies on the linear contact made by V.S. Covalschi and M. M. Saverin [4].

An analytical approach for the circular contact [5] proves, using the symmetrical properties of the integrals, that in the central plan *xOy* which is parallel with the direction of action of tangential forces, there exist a tangential tension $\tau_{yx}$ which reaches its extreme on the central axis *Ox* and is negative. The tensions $\tau_{xz}$ and $\tau_{yz}$ are cancelled. The normal tensions $\sigma_{xx}$, $\sigma_{yy}$ and $\sigma_{zz}$ are considered in a global context through hydrostatic pressure.

This paper addresses the state of tension produced in plans *xOz* and *yOz* by the tangential and radial forces applied to the contact with a specific direction on the circular contact area, and intents to specify the geometrical form of the gap and of the spin which influences the minimum thickness of the lubricant film. To keep things simple, the paper considers the operation of hydrodynamic bearings which is based on the principle of obtaining carrying capacity at the flow of lubricant through the two non-parallel surfaces, when the pressure in the film, which equilibrates the applied load, is created naturally by relative movement of surfaces.

## 2. Theoretical principles

To solve the problem, the Reynolds equation for short hydrodynamic bearing takes the following form:

$$\frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = 6\eta\omega \frac{\partial h}{\partial \theta} \qquad (1)$$

After applying the integral for an arbitrary angle of position $\theta$ we get the following expression:

$$p(z,\theta) = -\frac{3\eta h\omega\varepsilon}{J^2}\frac{\sin\theta}{(1+\varepsilon\cos\theta)^3}\left(z^2 - lz\right) \qquad (2)$$

In this case $J = R_2 - R_1$ - represents the radial gap in the bearing (fig. 2), and the film thickness has the following form:

$$h = J(1+\varepsilon\cos\theta) \qquad (3)$$

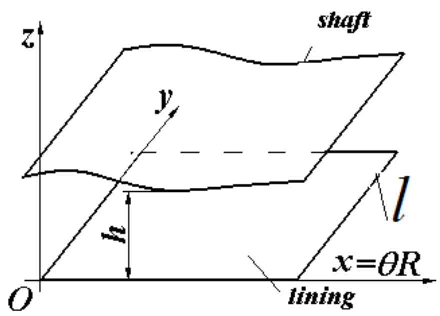


**Figure 1**

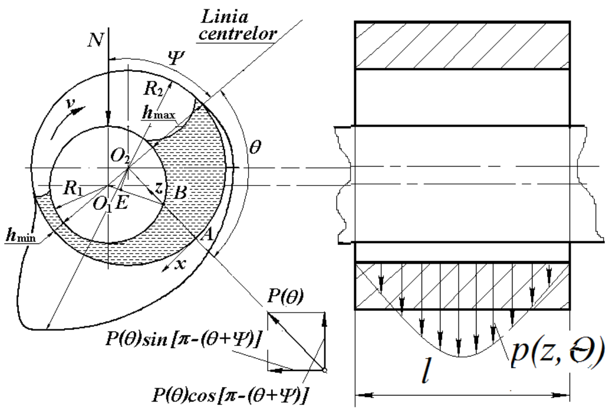


**Figure 2**

In order to establish and maintain an operating mode with liquid friction sliding, the size of the bearing and spindle need to be set considering the rigidity conditions that will exclude the mutual "hooking" of roughness of the spindle and bearing. This will ensure that the thickness of the lubricant film in the narrowest place (fig. 1), is bigger than the sum of the heights of the roughness of surfaces of the spindle - $H_1$ and bearing - $H_2$. Therefore, when calculating the relative gap in the case of radiated sliding bearings, the deformations of coupling elements [5] should be taken into account.

Since the level reached by the state of tension in a point of an elastic body stressed by a distributed load $p(z,\theta)$ according to (2) can be estimated through tensions, deformations or potential energy of deformation, we will next establish the relationships which will allow presenting the diagrams of the main tensions and displacements.

The normal tensions along the radial and circumferential direction in polar coordinates according to [7, 8] is written as

$$\sigma_{rr} = \frac{1}{r^3}\left(T - r^2\frac{\partial \Omega}{\partial z}\right)\cos\theta \ , \qquad (4)$$

$$\sigma_{\theta\theta} = -\frac{1}{r^3}\left\{T + r\frac{\partial}{\partial r}\left[2\mu\varphi + (1-2\mu)f\right]\right\}\cos\theta \ , \qquad (5)$$

and

$$\tau_{r\theta} = \frac{1}{r^3}T\sin\theta \ , \qquad (6)$$

where:

$$T = \frac{\partial \psi}{\partial z} - \left[2\mu\varphi + (1-2\mu)f\right] \ , \qquad (7)$$

$$\psi = \chi + z\varphi \ , \qquad (8)$$

$$\Omega = \frac{1}{r}\frac{\partial \psi}{\partial r} - \frac{\partial f}{\partial r} \ , \qquad (9)$$

for which $\chi$, $\gamma$ and $f$ - represent the tension functions.

To solve this problem, when the bearing is stressed on lateral surfaces with proportional loads $\cos\theta$ and $\sin\theta$, the tensions' function is chosen of the following form:

$$\chi = A\left(2z^2 - r^2\right)r^4 z \qquad (10)$$
$$\varphi = B\left(6z^2 - r^2\right)r^4$$
$$f = C\left(6z^2 - r^2\right)r^2$$

The tensions' function in polynomial form (2) satisfies the bi-harmonic condition.

Therefore, for solving the flat problem of tensions in bottomless thick-walled cylinder stressed only by non-symmetric internal pressures, the normal radial, circumferential as well as tangential tensions are written as:

$$\sigma_{rr} = p(z,\theta)\frac{r\left(a^2z^2 - l^2r^2\right)}{a^2\left(l^2 - z^2\right)}\cos\theta \qquad (13)$$

$$\sigma_{\theta\theta} = p(z,\theta)\times$$
$$\times\frac{168l^2rz^2 + 51l^2r^3 - 19a^2rz^2 - 6a^2r^3}{3a^2\left(l^2 - z^2\right)}\cos\theta \qquad (14)$$

$$\tau_{r\theta} = p(z,\theta)\times$$
$$\times\frac{16l^2rz^2 - 24l^2r^3 - 22a^2rz^2 + 3a^2r^3}{6a^3\left(l^2 - z^2\right)} \qquad (15)$$

Relationships (13 - 15) represent the flat state of tensions and allow to estimate the deformations of the bottomless thick-walled tube, that is, of the movement of points on the internal cylindrical surface which is stress non-symmetrically by internal pressures.

In order to calculate the movements of a specific point in the radial direction, the general solution for movements for a thick-walled cylinder when the load applied on the internal surface is proportional to $\sin\theta$ and $\cos\theta$ is taken as basis and takes the following form:

$$u_r = -\frac{1}{2G}\frac{1}{r^2}\left(T + r\frac{df}{dr}\cos\theta - u_0 ctg\,\theta\right) \qquad (16)$$

where

$$\frac{du_0}{dr} = \frac{1}{2G}\frac{1}{r^3}\left(T - r\frac{df}{dr}\right)\sin\theta \ ,$$

$$T = \frac{d\psi}{dz} - \left[2\mu\varphi + (1-2\mu)f\right] \ ,$$

$$\psi = \chi + z\varphi \qquad (19)$$

In the case of the non-stressed along the axial direction tube, for determining the movement of a point in the radial direction induced by the stress applied on the internal surface $p(z,\theta)$, the equation (16), when the mass forces are zero, is complemented with the following equations:

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}\frac{d\tau_{r\theta}}{d\theta} + \frac{d\sigma_{rr} - d\sigma_{\theta\theta}}{r} = 0 \ , \qquad (21)$$

$$\frac{d\tau_{r\theta}}{dr}+\frac{1}{r}\frac{d\tau_{\theta\theta}}{d\theta}+\frac{2\tau_{r\theta}}{r}=0, \qquad (22)$$

For the boundary conditions

$$\sigma_{rr}(r=a,\theta)=-p(z,\theta)\ , \qquad (23)$$

$$\sigma_{rr}\left(r=a,\theta=0^\circ\right)=0\ , \qquad (24)$$

$$u_r(r=a,z,\theta)=0\ , \qquad (25)$$

the constants *A*, *B*, and *C* are calculated.

$$A=\frac{6l^2-a^2}{36a^3\left(l^2-z^2\right)}p(z,\theta) \qquad (26)$$

$$B=\frac{6l^2-a^2}{18a^3\left(l^2-z^2\right)}p(z,\theta) \qquad (27)$$

$$C=\frac{15a^2-126l^2-24\mu l^2+4\mu a^2}{36a^3(1-2\mu)\left(l^2-z^2\right)}p(z,\theta) \qquad (28)$$

When substituting the values of constants obtained in (16) we get the relationship based on which we can calculate radial displacements for any point with the radius $r$ in the bottomless thick-walled cylinder stressed on the internal surface with the load $p(z,\theta)$ of the form:

$$u_r=-\frac{p(z,\theta)\left(b^2-r^2\right)\cos\theta}{24Ga^3\left(l^2-z^2\right)(1-2\mu)}\times \qquad (29)$$

$$\times\begin{bmatrix}\left(b^2-r^2\right)\left(15a^2-21\mu a^2-129l^2\right)+\\+\mu l^2\left(456z^2-64r^2-379b^2\right)-\\-a^2z^2\left(6-16\mu^2\right)\end{bmatrix}.$$

When the designer chooses to employ an optimisation study with multiple parameters, the obtained relationship can be used for the design of a sliding bearing when determining the main optimal requirements for a highly efficient and reliable functioning.

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