ON DIRECT COUPLING OF TWO SHAFTS. PART1: STRUCTURAL CONSIDERATION

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Abstract: *The paper aims to analyze the kinematics of a direct coupling of two shafts with fixed crossed axes. The first part of the paper deals with the structural issues, signifying that the simplest solution for transmitting motion between two shafts is the direct coupling through an installation of class 1 pair. The need to transmit continuously movement led to the final option, that the contact should be made between two lines, one parallel to one of the shafts and the other perpendicular to the second shaft.*

Keywords: *crossed shafts, structural analysis, linkages*

1. General considerations

In the engineering practice often occurs the problem of transmission of the rotational motion between two shafts 1 and 2 with fixed axes, which is achieved by coupling them to the ground 0 by using two revolute pairs, A and B.

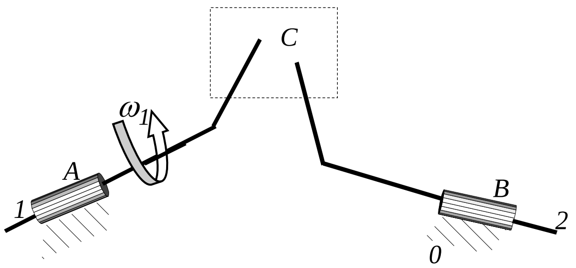

Figure 1 *General coupling of two shafts*

The problem admits a multitude of solutions, the coupling kinematic chain, noted with C, being named *coupling*. The structure and the dimensional characteristics of the coupling kinematic chain depend essentially on the following factors:

a) the relative position of the axes of the two rotational couplings: parallel, concurrent or the general case of crossed axes;
b) the quality of the transmission of the motion described by the transmission ratio [1]:

$$i_{12} = \frac{\omega_1}{\omega_2}, \tag{1}$$

being distinguished two cases here:
a) constant transmission ratio (homokinetic transmissions)
b) variable transmission ratio.

Lately have emerged new coupling solutions using flexible elements: corrugated tubes, elastic tubes and bars which led to the emergence of compliant mechanisms [2].

2. Possible structural solutions

The general structure of a kinematic chain includes links and pairs (the linkages between them). The main criterion for the classification of the kinematic pairs is their class, equal to the number of simple movements which are canceled. As a rigid in general motion has 6 degrees of freedom, relative to a Cartesian coordinate system, the class of pair c_k, can take these values: $k = 1 \div 5$. The degree of freedom of a kinematical chain, defined as the number of independent scalar parameters which univocal define the position of kinematical chain, is given by the relation:

$$L = 6n - \sum_{k=1}^{5} kc_k \tag{2}$$

In engineering applications there is often met the situation when a number of f constraints is imposed to all the elements of the chain and thus it can be considered that the motion of the chain runs in a space with $6 - f$ dimensions; the relation 2 takes the form:

$$L_f = (6 - f)n - \sum_{k=f+1}^{5} (k - f)c_k \tag{3}$$

The mechanism is a particular case of closed kinematic chain that has a fixed element named ground and the motion of any element is analyzed with respect to it. In this case, the degree of mobility is defined as:

$$M_f = (6 - f)(n - 1) - \sum_{k=f+1}^{5} (k - f)c_k \tag{4}$$

In the situation of two shafts, it is considered the general case of 0 family chain where the kinematic chain of coupling is composed of n elements and c_k k class couplings. In addition the formula 4 should also include three elements: the ground, the two shafts and the connecting couplings between the shafts and the ground. In the end the degree of mobility of the mechanism must be $M_0 = 1$, so the relation 4 become:

$$1 = 6(n + 2) - \sum_{k=1}^{5} kc_k - 2 \cdot 5 \tag{5}$$

3. Structural solutions for direct coupling

In this case the two shafts make direct contact so that the number of elements of the kinematic chain coupling is $n = 0$ and equation 5 become:

$$1 = 6 \cdot 2 - \sum_{k=1}^{5} kc_k - 2 \cdot 5 \tag{6}$$

with an immediate consequence:

$$\sum_{k=1}^{5} kc_k = 1 \tag{7}$$

The single unique solution being:

$$c_1 = 1, c_k = 0, k > 1 \tag{8}$$

Equation 8 shows that for direct coupling of the two shafts they must achieve a point contact.

The point contact can be achieved in four ways:

a) The contact of two non-conforming surfaces, like mechanisms with gears and mechanisms with the curved face follower where the follower and cam axes are not parallel, Fig. 2, [3], and Fig. 3 [4].

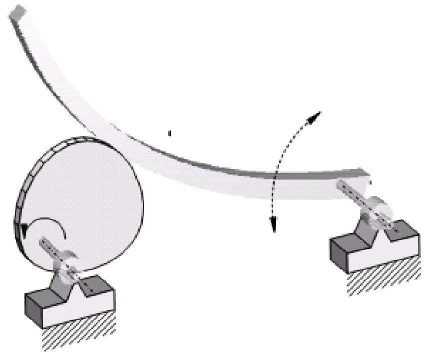

Figure 2 *Spatial mechanism with rotating cam and oscillating curved face follower [3]*

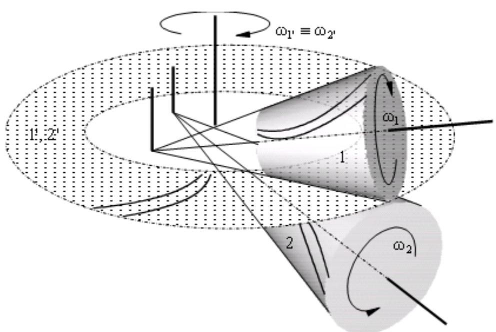

Figure 3 *Hypoid gear pair [4]*

When it is necessary that the trasmission ratio to be constant the solution is the use of gears, represented by the crossed axes gears, with cylindrical, hypoid or worm and worm gear, Fig. 4 [5].

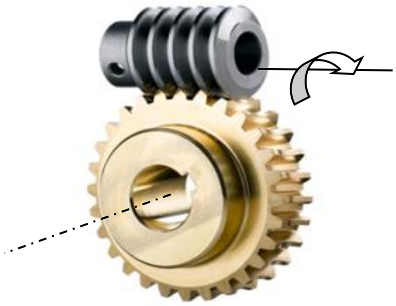

Figure 4 *Worm and worm gear pair [5]*

b) Contact between a surface and a curve; as an example it can be considered a spatial mechanism with rotating cam and oscillating rectilinear cam follower, Fig. 5.

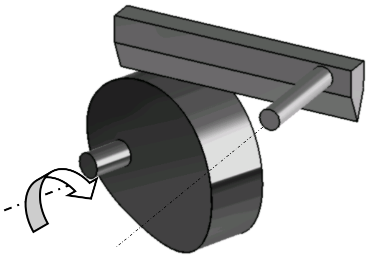

Figure 5 *Contact between a surface and a curve*

c) contact between a point and a surface, as shown in Fig. 6.

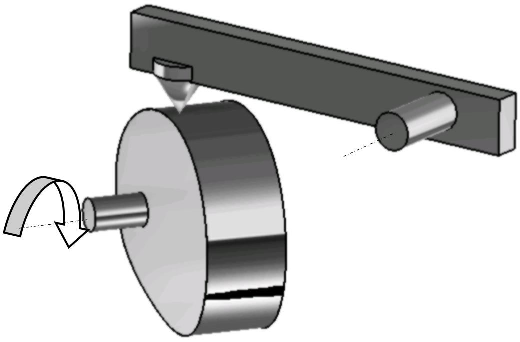

Figure 6 *Contact between a point and a surface*

d) Contact between two curves, presented in Fig. 7. During operation the two curves depict rotational surfaces coaxial with the axes rotating pairs. In practical cases the two curves are straight lines so that during operation they depict rotating conical surfaces and hence the conclusion that the contact point will be on the curve of intersection of the two conical sheets, Fig. 8.

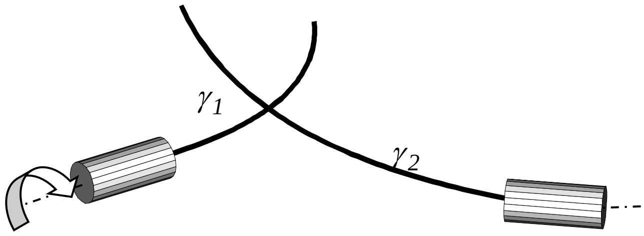

Figure 7 *Direct contact of two curves*

A rational construction of the transmission means that the curves of intersection must be closed so the transmission will have finite dimensions, Fig. 8.

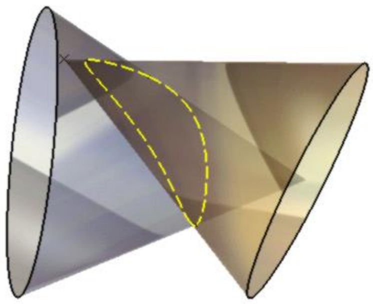

Figure 8 *The intersection of two conical sheets*

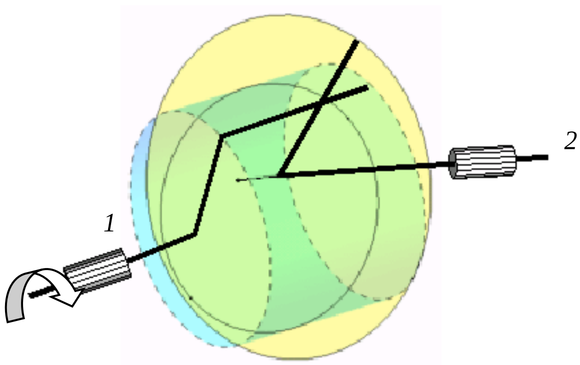

Figure 9 *The intersection between a cylinder and a plane*

As a practical solution to ensure that the conditions of closing the curve on which the contact point between the two curves moves, it is used a limit case of the general situation presented in Figure 7. More specifically, it considers the case when one of the two cones is turning into a cylinder, while the other degenerates into a plan, Fig. 9. The intersection between a cylinder and a plane is always an ellipse, unless the plane is parallel to the axis of the cylinder, when the intersection between the two surfaces is represented by two parallel straight lines, real or imaginary, issue determined by the relation between the radius of the cylinder and the distance from the plane to the axis of the cylinder. For kinematic analysis, it is useful to replace the actual mechanism with a mechanism with only class 4 cylindrical lower pairs, which may be in particular forms of class 5 revolute and prismatic pairs. Considering that the kinematic chain for coupling consists only of kinematic elements connected by class 5 pairs and the link with the two shafts is realized through class 5 pair, if n' denotes the number of elements of the chain and c'_5 the number of class 5 pairs, the equation 2 for the whole mechanism becomes:

$$1 = 6(n'+2) - 5'c_5 - 2 \cdot 5 \qquad (9)$$

which can be written:

$$6n' - 5c'_5 = -1 \qquad (10)$$

The simplest non-trivial solution of equation 10 is:

$$n' = 4; \; c'_5 = 5 \qquad (11)$$

Equation 11 is consistent with the fact that the simplest spatial mechanism in whose structure there are only class 5 pairs consists of seven binary elements linked through seven class 5 pairs.

Figure 10 *RCCC mechanism [6]*

Another possibility for coupling two shafts with kinematic chains with lower pairs, useful in applications, is the development of a RCCC type mechanism [6] in which the revolute pair acts as the driving pair, while the driven shaft is part of a cylindrical pair. The advantage of this coupling solution consists in the existence of an analytical solution due to Yang [7] for all the movements between the joints of the mechanism.

4. Conclusions

The paper aims to make a kinematic analysis of a solution by direct coupling of two shafts fixed crossed axis. The first part of the paper deals with structural aspects, pointing out that the simplest solution for transmission of movement between the two shafts is the direct coupling via a class 1 pair. Since the practical materialization of class 1 pair is possible in several ways, using different geometry, following an analysis it will be chosen that the connection to be achieved by contact of two curves. Besides, the need for continuously transmitting of the motion led to the final option that the contact to be made between two straight lines, one parallel to one of the shafts and the other perpendicular to the second.

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