THE NORMAL CONTACT OF FINITE LENGTH COATED CYLINDERS

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Abstract: *The competent design of tribological systems involving coated bodies requires numerical methods. The elastic response of the coated body to surface load has only been expressed analytically in the frequency domain. This encourages the use of spectral techniques such as the Discrete Convolution Fast Fourier Transform (DCFFT) method that achieves efficient convolution computation in the frequency domain. The displacement of coated bodies can thus be evaluated by DCFFT provided the needed influence coefficients are accurately derived. This paper explores a method for conversion of the required influence coefficients from the frequency response functions available in the literature. The conversion procedure is benchmarked against the solution for the spherical indentation of a coated half-space. The program is then applied to the finite length line contact, in which the stress singularity related to end effects is relieved by roller profiling. The presented contact scenarios prove the method ability to simulate the finite length line contact of coated materials.*

Keywords: *numerical simulation, coating, finite length line contact, end effect*

1. Introduction

The tribological performance of various elements undergoing contact loadings can be increased via protective coatings that provide low friction, high wear resistance and long fatigue life. The competent coating design relies on the knowledge of the stresses generated in the coated system under load. The complex mathematical models that govern the contact process, many without analytical solution, suggest the implementation of numerical analysis.

The most important advances in the modelling of layered elastic materials have been attained by Fourier analysis, allowing for explicit relations describing the elastic response of the layered body in the frequency domain. From a computational point of view, considering that the displacement response of the elastic multilayered body can be expressed as a convolution product, it is convenient to perform the contact analyses in the Fourier transform domain.

The mathematical modelling of the contact between layered elastic materials, pioneered by Burminster [1], was advanced by the works of O'Sullivan and King [2], Ju and Farris [3], and Nogi and Kato [4]. The latter authors combined the fast Fourier transform (FFT) with the conjugate gradient method (CGM) to obtain the solution of contact problems for both homogenous and layered solids. The periodicity error associated with the application of FFT to non-periodic problems was discussed and ameliorated by Polonsky and Keer [5,6].

An important computational breakthrough was achieved [7,8] with the method of discrete convolution and FFT (DCFFT), which completely avoids any error except for the discretization error.

This paper aims to advance a DCFFT-based simulation technique for contact analyses applicable to bodies with prescribed geometry, starting from frequency response functions (FRFs) existing in the literature [2,4,9], describing in the frequency domain the elastic response of the layered half-space to a unit normal load.

2. Contact model

The employed contact model [10,11] is based on three type of equations: (1) the equation of the surface of deformation between the two bodies, (2) the boundary conditions, and (3) the static equilibrium, the latter including both force and torque equations in order to properly simulate the force eccentricity effect.

The aforementioned contact model is difficult to solve because neither the contact area, nor the pressure distribution are known in advance. The elastic displacement response of the multilayered contacting body is of paramount importance to the model solution.

The numerical treatment of this contact model involves [11] the spatial discretization of the contact surface using a rectangular uniform mesh established in the common plane of contact, on which all problem parameters are assumed piecewise-constant. With this approach, integration of arbitrary functions over arbitrary domains is substituted by summation, which can be performed regardless of the prescribed input. In this manner, the notation of problem parameters can make use of the discrete indexes of the discrete surface patches, i.e. $p(i, j)$ is the uniform pressure predicted for the patch (i, j) of the surface grid.

The displacement response to a unit normal force is available for the elastic and homogenous half-space as the Boussinesq fundamental solution [12]. The latter can be used to derive, by superposition of effects, the half-space response to a prescribed load. The counterpart of the Boussinesq solution for the layered material has only been derived in the frequency domain, i.e., the FRF. This encourages the use of the DCFFT technique that transfers the convolution computation in the frequency domain, to achieve increased computational efficiency. The DCFFT requires the influence coefficients (ICs), obtained by the integration of the Boussinesq solution over each discrete element of the mesh. The required ICs are analytical solutions (in the space domain) of displacement or stresses generated by unit surface load, computed in a finite computational domain expected to include the contact area.

For an uncoated elastic contact problem, the normal displacement is expressed as the cyclic convolution [7]:

$$u_i = \sum_{j=1}^{N} p_j \cdot C_{i-j+N\cdot H(j-i)}, \quad i = 1, \ldots, N, \quad (1)$$

where $H(x)$ denotes the Heaviside unit step function, and C_i is the influence coefficients series, i.e. the contribution of a unit pressure acting in the cell i to the displacement measured in origin.

Analytical derivation of the ICs for the homogenous, elastic and isotropic half-space is readily available, e.g. the displacement [13] induced by a uniform pressure applied on a rectangular patch. However, for the layered material, the computation of the ICs should be performed based on solutions (FRFs) expressed in the frequency domain only. The following section explores a numerical technique to obtain the required ICs in the space domain by conversion from the frequency domain via inverse FFT.

Once a computational method for the displacement induced in layered materials by a prescribed load is available, the solution of the contact problem of coated bodies can be obtained in the same manner as its counterpart for homogenous bodies. The employed solver is similar to the one originally advanced by Polonsky and Keer [11] for the elastic contact of rough surfaces, in which the multilevel multisummation (MLMS) sequence for displacement computation is replaced by the DCFFT technique.

3. Conversion of ICs from FRFs

Although the three dimensional contact problem involves two-dimensional parameters, e.g. pressure or surface displacement, the algorithm steps will be written in one dimension for simplicity. The technique to derive in the space domain the required ICs by inverse FFT applied to the discrete series computed from the FRFs is described below:

1. Establish the target computational domain, i.e. the data interval Δ, the number of nodes N and the refinement factor 2χ, assuring minimization of the aliasing phenomenon. The goal is to compute the influence coefficients series $\{C_i\}_{2N}$ in a domain of size $2N\Delta$, i.e., double the problem physical domain (as requested by the DCFFT).

2. Compute the coordinates of the nodes in the spectral mesh, obtaining a series of discrete frequencies $\{m_i\}_{2\chi N}$, $i=1,\mathrm{K},2\chi N$. The size of the spectral patch is $2\pi/(N\Delta)$.

3. Compute the values of the FRF in the latter nodes, thus obtaining a complex series $\{\hat{g}_i\}_{2\chi N}$, $i=1,\mathrm{K},2\chi N$.

4. Rearrange the latter series in wrap around order, i.e., the terms related to negative frequencies are repositioned after the other ones. Obtain a new series $\{\hat{f}_i\}_{2\chi N}$.

5. Apply inverse FFT to the latter series to obtain the ICs $\{C_i\}_{2\chi N}$, and retain only the middle $2N$ terms as algorithm output.

The resulting series $\{C_i\}_{2N}$ of influence coefficients is the one needed in the DCFFT algorithm, assuring computation of displacement in a target domain of size $N\Delta$.

The 2D FRF required for the computation of displacement induced by pressure in a coated material can be expressed as [4]:

$$\hat{u}(m,n) = -(1-\nu_1)(1+4\alpha h\kappa\theta-\lambda\kappa\theta^2)\alpha R/G_1\,, \qquad (2)$$

where h is the layer thickness, $\alpha=\sqrt{m^2+n^2}$, $\mu=G_1/G_2$, $\theta=e^{-2\alpha h}$, $\kappa=(\mu-1)/(\mu+3-4\nu_1)$, $\lambda=1-4(1-\nu_1)/[1+\mu(3-4\nu_2)]$, and $R=-\alpha^{-2}/[1+(\lambda+\kappa+4\kappa\alpha^2h^2)\theta+\lambda\kappa\theta^2]$. The shear moduli and the Poisson's ratios for the layer and the substrate are denoted by G_i and ν_i respectively, with $i=1$ for the layer and $i=2$ for the substrate.

4. Contact geometry

Infinite pressure is predicted theoretically at the sharp edge of the shorter contacting body in a finite length line contact. This is not the case in practical applications, due to plastic yielding or manufacturing limitations. However, important pressure risers may arise, compromising the load carrying capacity of the contact. Reduction of these end effects by roller profiling was studied numerically by these authors [14,15]. The gradient of pressure increases abruptly, and the peak value is very sensitive to discretization step. In fact, if the contacting shorter cylinder is not rounded and the material is assumed to behave purely elastic, a singularity in the contact stress is expected. The latter singularity can only be assessed numerically in an approximate manner, as the numerical technique employs averaged parameters. In this paper, a rounding radius R_a is introduced to diminish the end effects. The rounded part is assumed tangent to the linear portion of the roller generatrix:

$$z(x,y)=\begin{cases} R-\sqrt{R^2-y^2}, & |x|\le L \\ R-\sqrt{\left[R_a-R-\sqrt{R_a^2-(x-L\cdot\mathrm{sgn}(x))^2}\,\right]^2-y^2}, & |x|>L \end{cases} \qquad (3)$$

where L denotes the half-length of the linear portion of the roller generatrix, and R the radius of the cylinder.

5. Results and discussions

The newly advanced computer program was validated against the solution of the coated spherical contact presented in the literature [4,9]. The surface displacement respons of the layered solid is obtained with the aid of the DCFFT with the ICs obtained by conversion from the FRFs, as described in a previous section. The Poisson's ratios of both coating and substrate were fixed at 0.3, as well as the Young modulus of the substrate, E_2, while that of the coating, denoted by E_1, was varied. The contact of a rigid sphere with the uncoated homogenous half-space (i.e., the Hertz contact with $E_1 = E_2$) is taken as reference. Figure 1 depicts the predicted pressure distribution normalized by the maximum Hertzian pressure p_H. The radial coordinate and the layer thickness are normalized by the Hertz contact radius a_H.

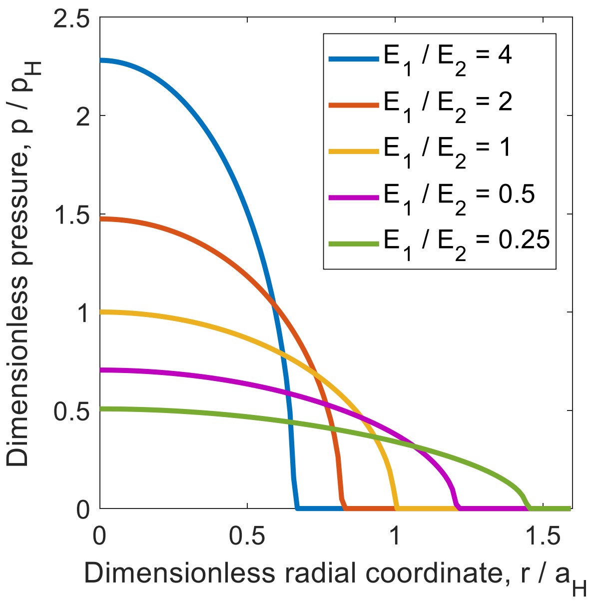

Figure 1: Spherical indentation of a coated half-space, $h/a_H = 1$

The finite length line contact of coated materials is studied by pressing a rigid roller whose contact geometry is described by equation (3), into a coated half-space, with a normal force $W = 1\,kN$. The roller dimensions are taken as suggested in [14]: $2L = 19.13\,mm$, $R_a = 7500\,mm$, $R = 5\,mm$. The Young modulus of the half-space is kept constant, $E_2 = 210\,GPa$, while that of the coating is varied. The coating thickness is fixed at $h = 0.04R$, matching closely the half-width of the contact area in the central region for the case $E_1 = E_2$. Pressure profiles in the planes $y = 0$ and $x = 0$, for different elastic mismatches between the coating and the substrate, are presented in figures 2 and 3, respectively, whereas a typical 3D pressure distribution is displayed in figure 4.

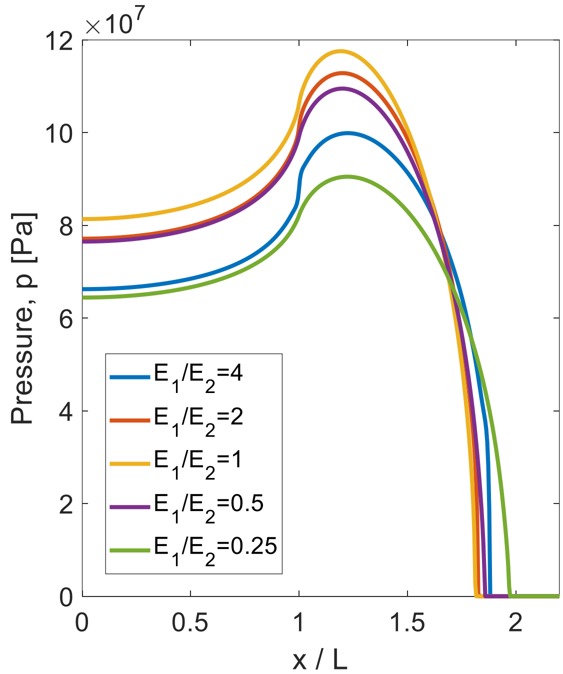

Figure 2: Longitudinal pressure profiles

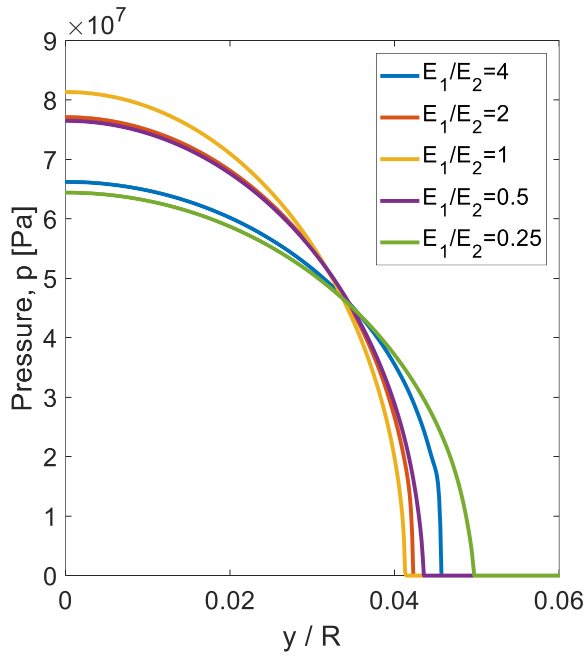

Figure 3: Transversal pressure profiles

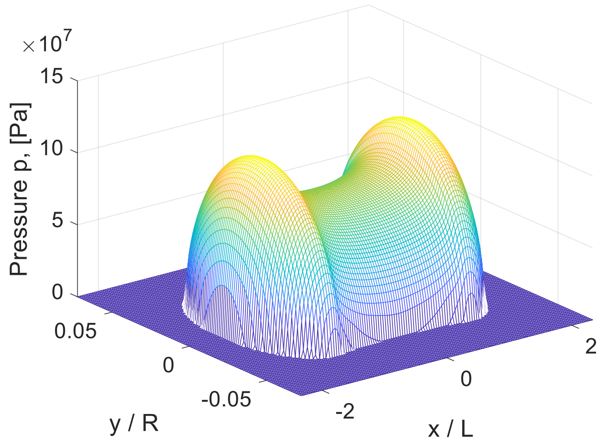

Figure 4: 3D pressure distribution, $E_1 = E_2$

The numerical program can also predict the finite length contact scenario involving a force eccentricity. Without losing generality, a dimensionless eccentricity $e/L = 0.3$ is assumed in the direction of the *x*-axis. The predicted longitudinal pressure profiles is depicted in figure 5, and the 3D pressure distribution for the case $E_1 = E_2$ in figure 6.

The numerical simulations suggest that the half-space coating lead to a diminishing in intensity of the pressure riser related to the end effect, accompanied by an increase in the contact area.

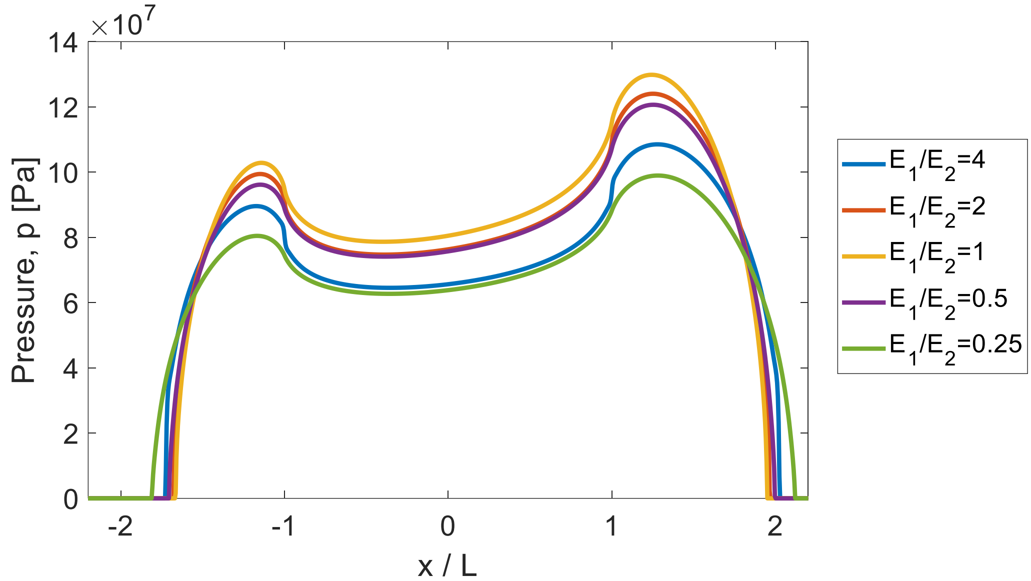

Figure 5: Longitudinal pressure profiles, eccentric loading

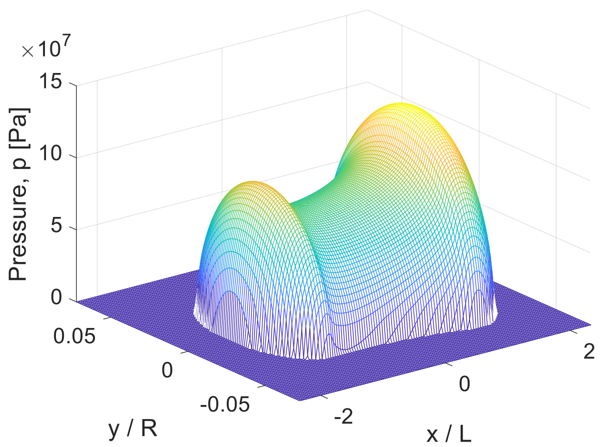

Figure 6: 3D pressure distribution in eccentric loading, $E_1 = E_2$

6. Conclusions

The finite length line contact involving coated bodies is simulated in this paper by combining a robust solver for the frictionless normal contact with the DCFFT method for displacement calculation in the frequency domain. As the response of the coated body is only known in the frequency domain, a conversion procedure of the influence coefficients required by the DCFFT technique is advanced.

Comparison with results from the literature for the spherical indentation of a coated half-space provides method validation and sanctions the computer program.

The singularity in contact stresses related to end effects is avoided by partial crowning of the roller. This assumption assures model compatibility with practical applications and sanctions the use of numerical methods.

The numerical technique is used to predict pressure distribution and contact area for various elastic mismatches between the coating and the substrate. The numerical examples prove the method ability to address the competent design of coated systems.

7. Acknowledgement

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